Sliding Basis Optimization for Heterogeneous Material Design

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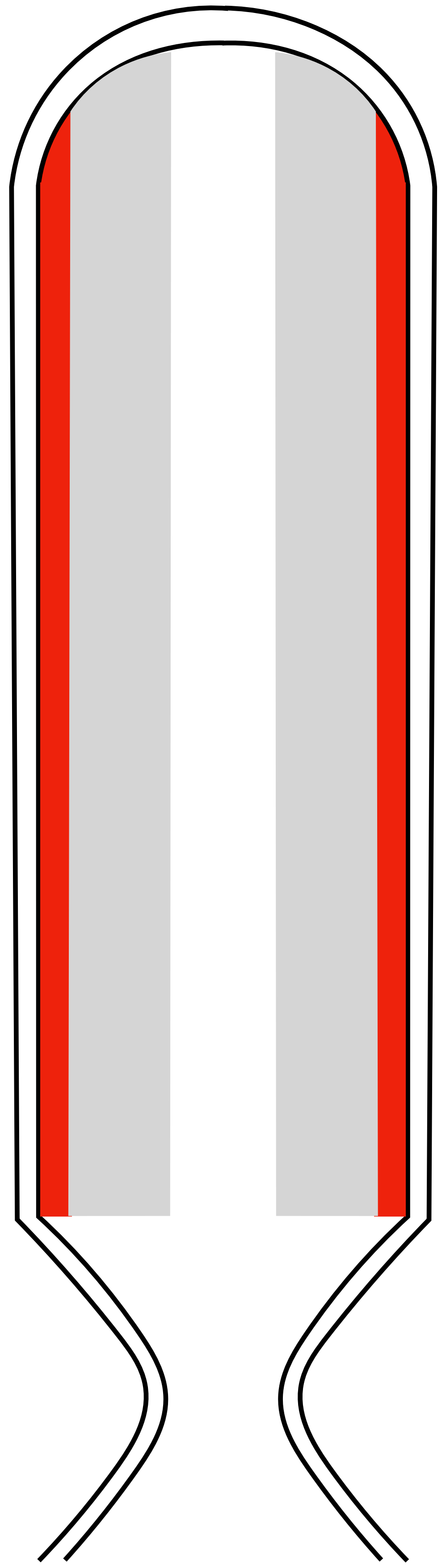
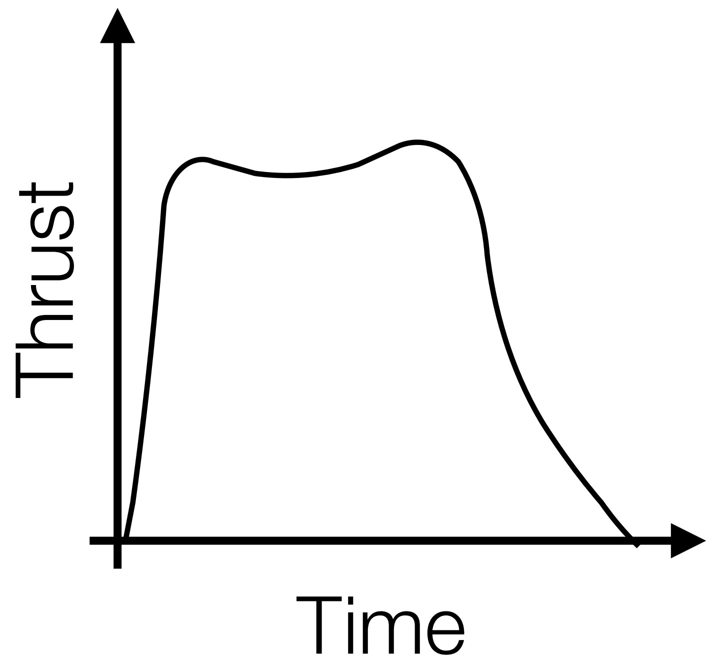
Palo Alto Research Center

Space Shuttle launch image courtesy **NASA.gov**

Solid Rocket Propellant Design

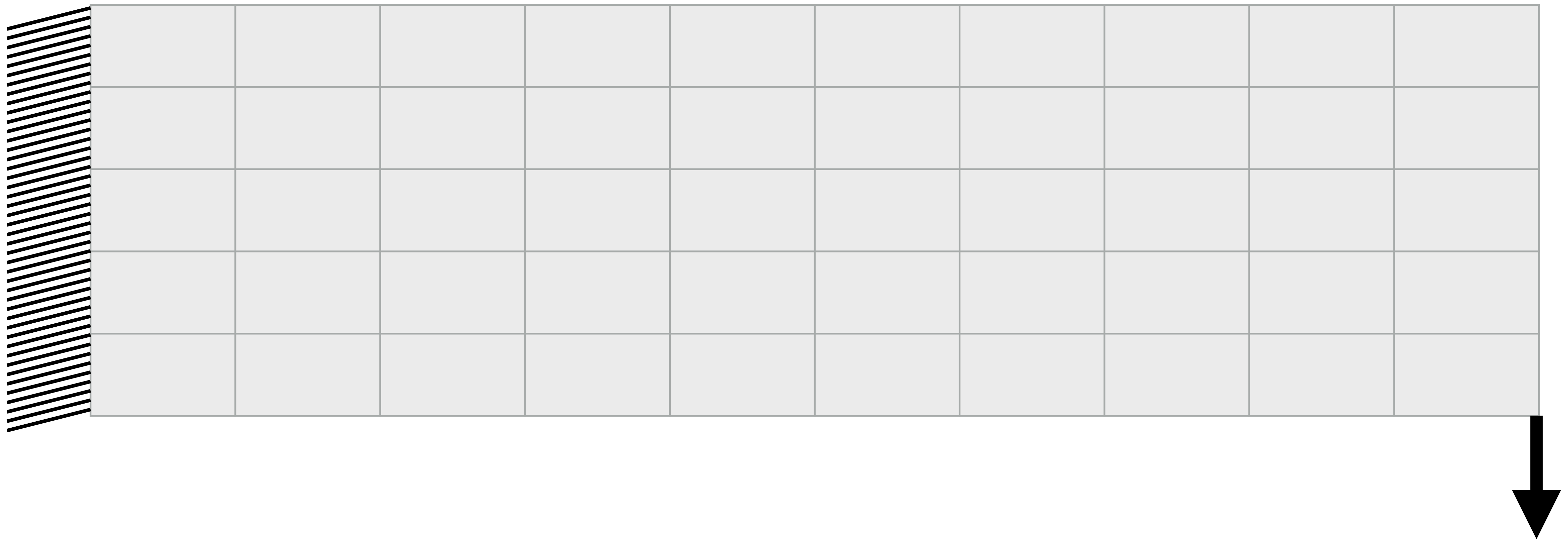




Geometry design with single material is not enough to achieve both desired thrust and eliminate insulation !!

General Material Design Optimization Pipeline

$$\begin{aligned} \min_{\boldsymbol{\mathcal{F}}} \quad & f(\boldsymbol{\mathcal{F}}) \\ \text{s.t.} \quad & g_i(\boldsymbol{\mathcal{F}}) \leq 0 \end{aligned}$$



$$\Phi(\boldsymbol{\mathcal{F}}) = 0$$

Costly analysis!

Complex physics!

Maybe black box!

Difficult to derive analytical gradients

Numerical gradients are not practical for large scale problems

Model reduction approach to reduce number of optimization variables

$$\begin{aligned}\min_{\boldsymbol{\mathcal{F}}} \quad & f(\boldsymbol{\mathcal{F}})\\ \text{s.t.} \quad & g_i(\boldsymbol{\mathcal{F}}) \leq 0\end{aligned}$$

$$\Phi(\boldsymbol{\mathcal{F}}) = 0$$

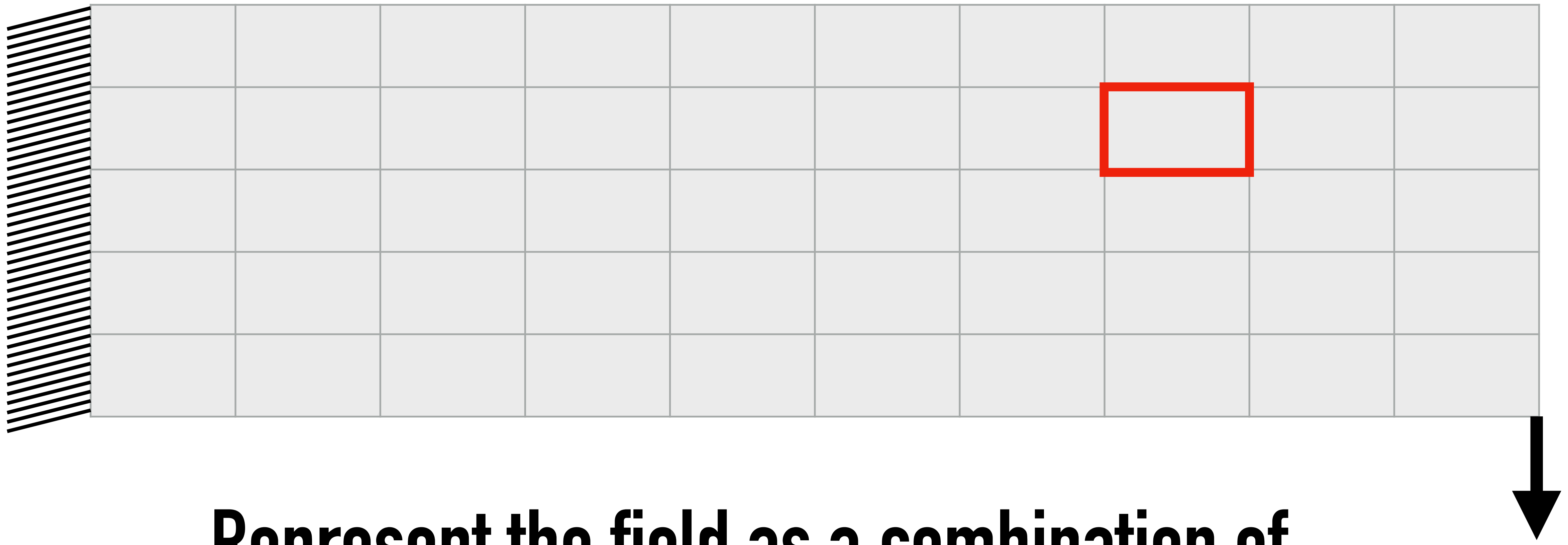
thousand-millions

➡

$$\begin{aligned}\min_{\boldsymbol{w}} \quad & f(\boldsymbol{w})\\ \text{s.t.} \quad & g_i(\boldsymbol{w}) \leq 0\end{aligned}$$

$$\Phi(\boldsymbol{w}) = 0$$

tens-hundreds



Represent the field as a combination of small set of basis functions!

$$\boldsymbol{\mathcal{F}} = \boldsymbol{B}\boldsymbol{w}$$

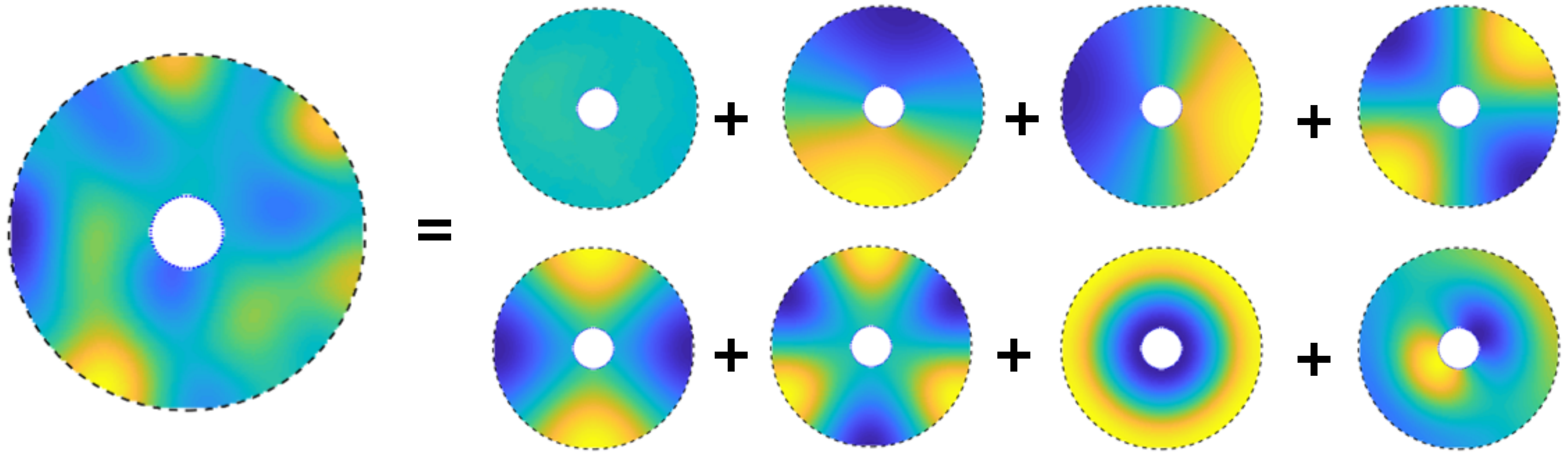
Reduce number of design variables without compromising analysis quality!

Representing Material Distributions Using Shape Harmonics

- Fourier expansion of signal into harmonics (sin + cos functions) can be applied to manifold decomposition
- Fourier bases are eigenfunctions of the Laplacian on the unit interval
- Spherical harmonics are eigenfunctions of the Laplacian on the sphere

$$\lambda_i \boldsymbol{e_i} = \boldsymbol{\mathcal{L} e_i} \quad \boldsymbol{B} = [\boldsymbol{e_1}, \boldsymbol{e_2}, ..., \boldsymbol{e_k}]$$

- Idea can be generalized to harmonics over any manifold
- Weighted sum of manifold harmonic basis can be used to describe shape and material in a ‘frequency domain’



$$\mathcal{F} = \boldsymbol{B w}$$

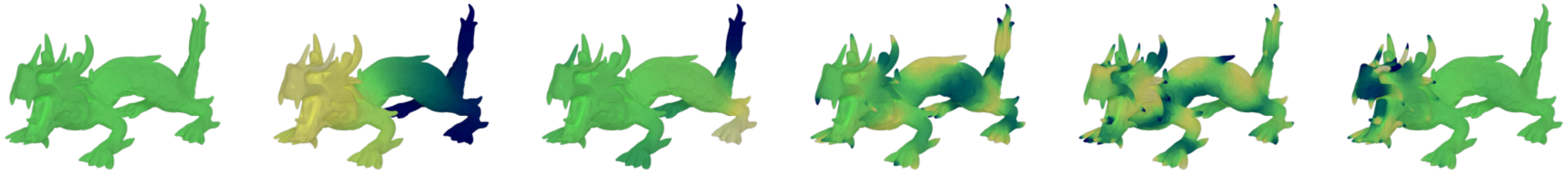
10k quad mesh **8 weights**

We represent a field with fewer parameters using Laplacian basis

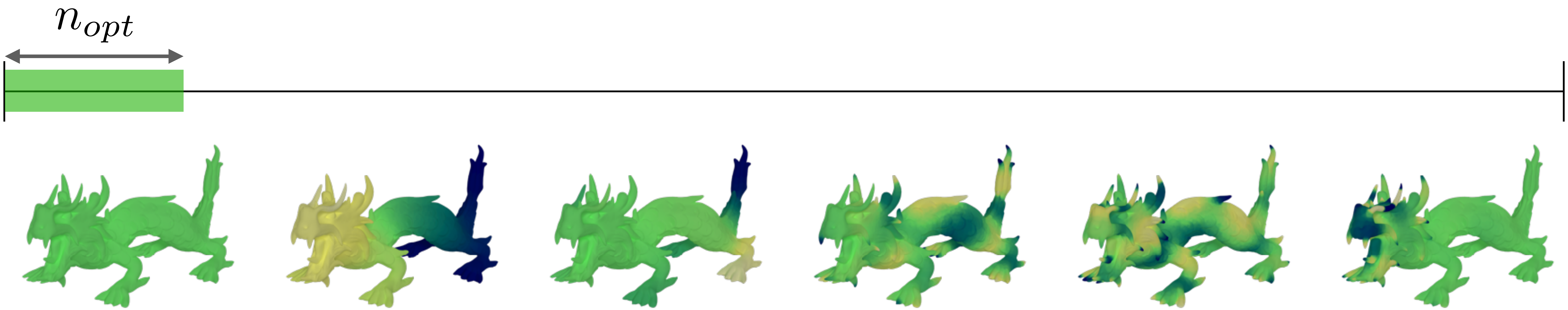
Key Observation

low frequency *high frequency*

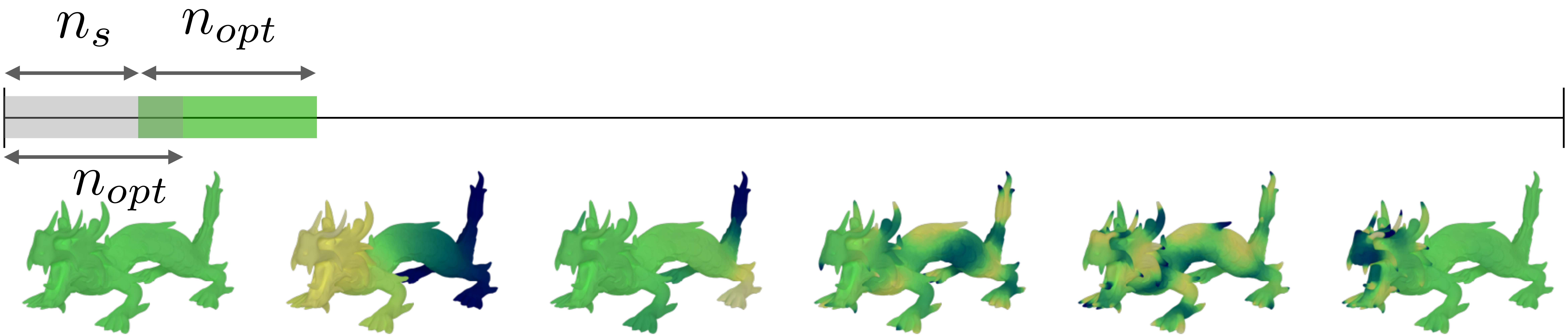
basis functions



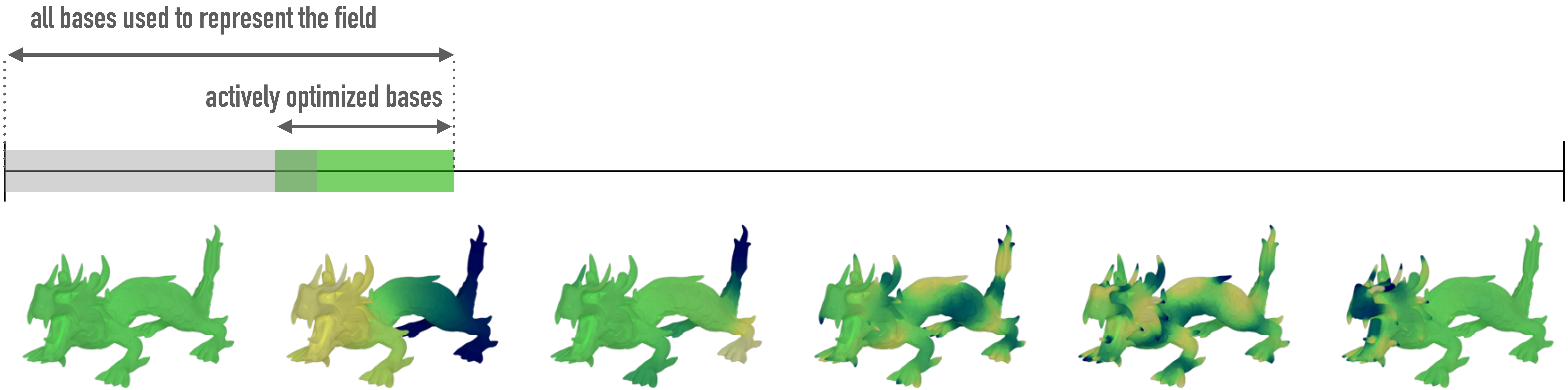
Sliding Basis Optimization



Sliding Basis Optimization

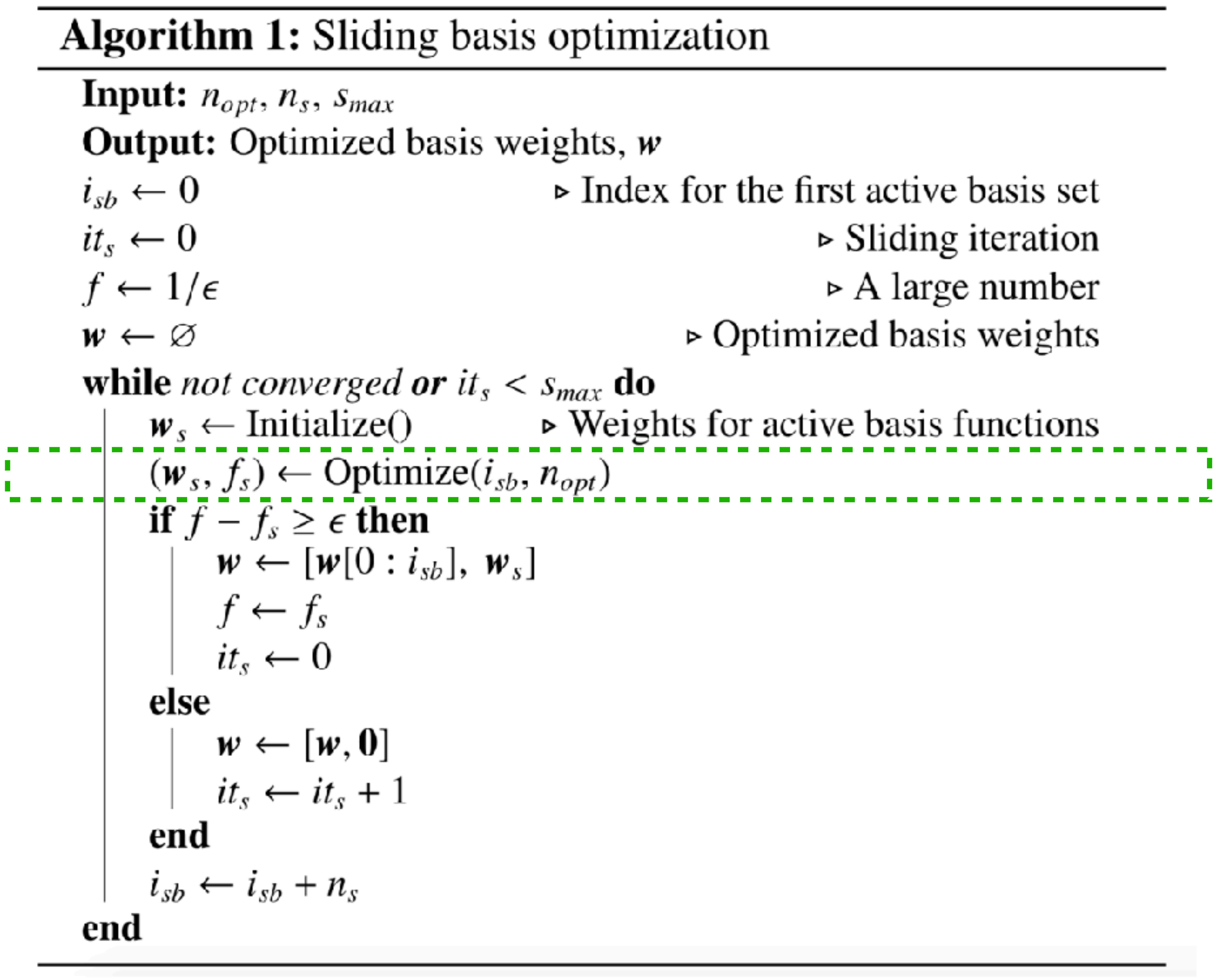


Sliding Basis Optimization



We usually get convergence using only a small set of basis functions!

Sliding basis optimization is a top level framework that works with existing optimization methods



Algorithm 1: Sliding basis optimization

```
Input: $n_{opt}$, $n_s$, $s_{max}$
Output: Optimized basis weights, $\boldsymbol{w}$
$i_{sb} \leftarrow 0$                                  ▹ Index for the first active basis set
$it_s \leftarrow 0$                                    ▹ Sliding iteration
$f \leftarrow 1/\epsilon$                              ▹ A large number
$\boldsymbol{w} \leftarrow \varnothing$                ▹ Optimized basis weights
while not converged or $it_s < s_{max}$ do
    $\boldsymbol{w}_s \leftarrow$ Initialize()         ▹ Weights for active basis functions
    $(\boldsymbol{w}_s, f_s) \leftarrow$ Optimize($i_{sb}$, $n_{opt}$)
    if $f - f_s \geq \epsilon$ then
        $\boldsymbol{w} \leftarrow [\boldsymbol{w}[0 : i_{sb}],\ \boldsymbol{w}_s]$
        $f \leftarrow f_s$
        $it_s \leftarrow 0$
    else
        $\boldsymbol{w} \leftarrow [\boldsymbol{w}, \boldsymbol{0}]$
        $it_s \leftarrow it_s + 1$
    end
    $i_{sb} \leftarrow i_{sb} + n_s$
end
```

This optimize step can be implemented using general nonlinear optimizers

Sliding basis optimization speeds up differentiable problems, too

$$\begin{aligned} \min_{\boldsymbol{w}} \quad & f(\boldsymbol{w}) \\ \text{s.t.} \quad & g_i(\boldsymbol{w}) \le 0 \end{aligned}$$

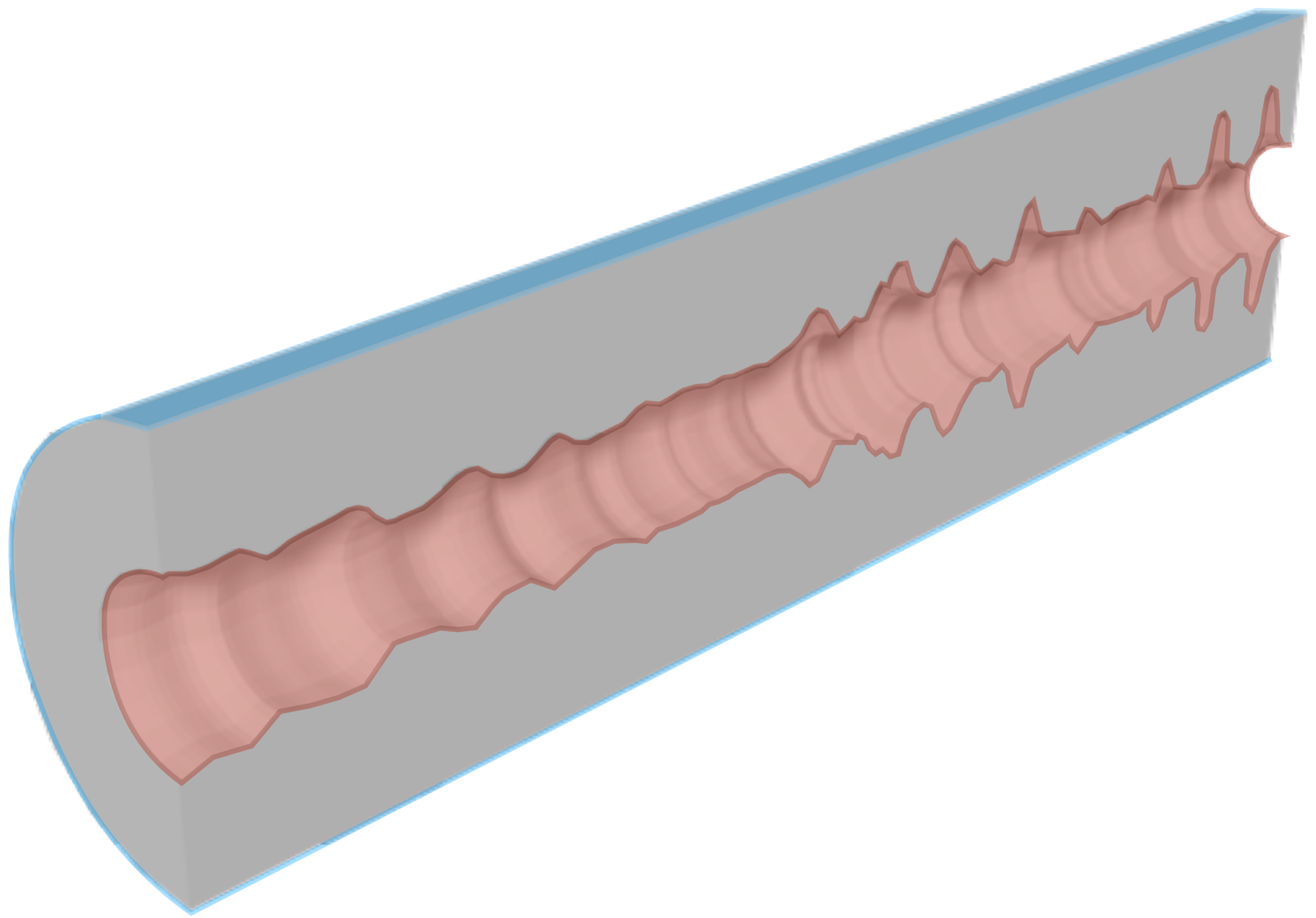
If gradients are derived
for the full resolution

$$\frac{\partial f}{\partial \boldsymbol{w}} = \boxed{\frac{\partial f}{\partial \boldsymbol{\mathcal{F}}}} \frac{\partial \boldsymbol{\mathcal{F}}}{\partial \boldsymbol{w}}$$

Gradients w.r.t. basis weights can be found
through simple matrix multiplication with B

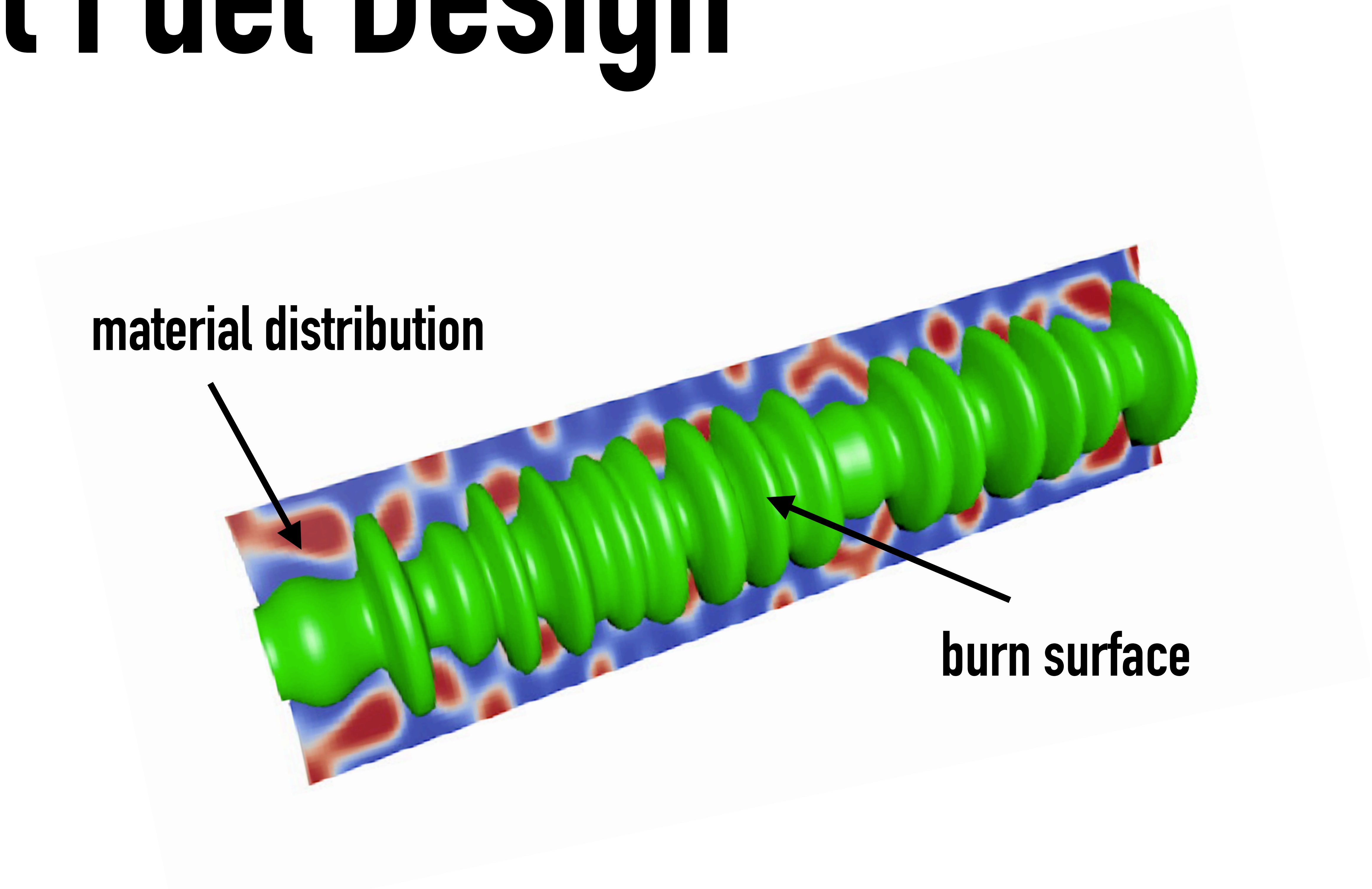
$$\boldsymbol{\mathcal{F}} = \boldsymbol{B}\boldsymbol{w}$$

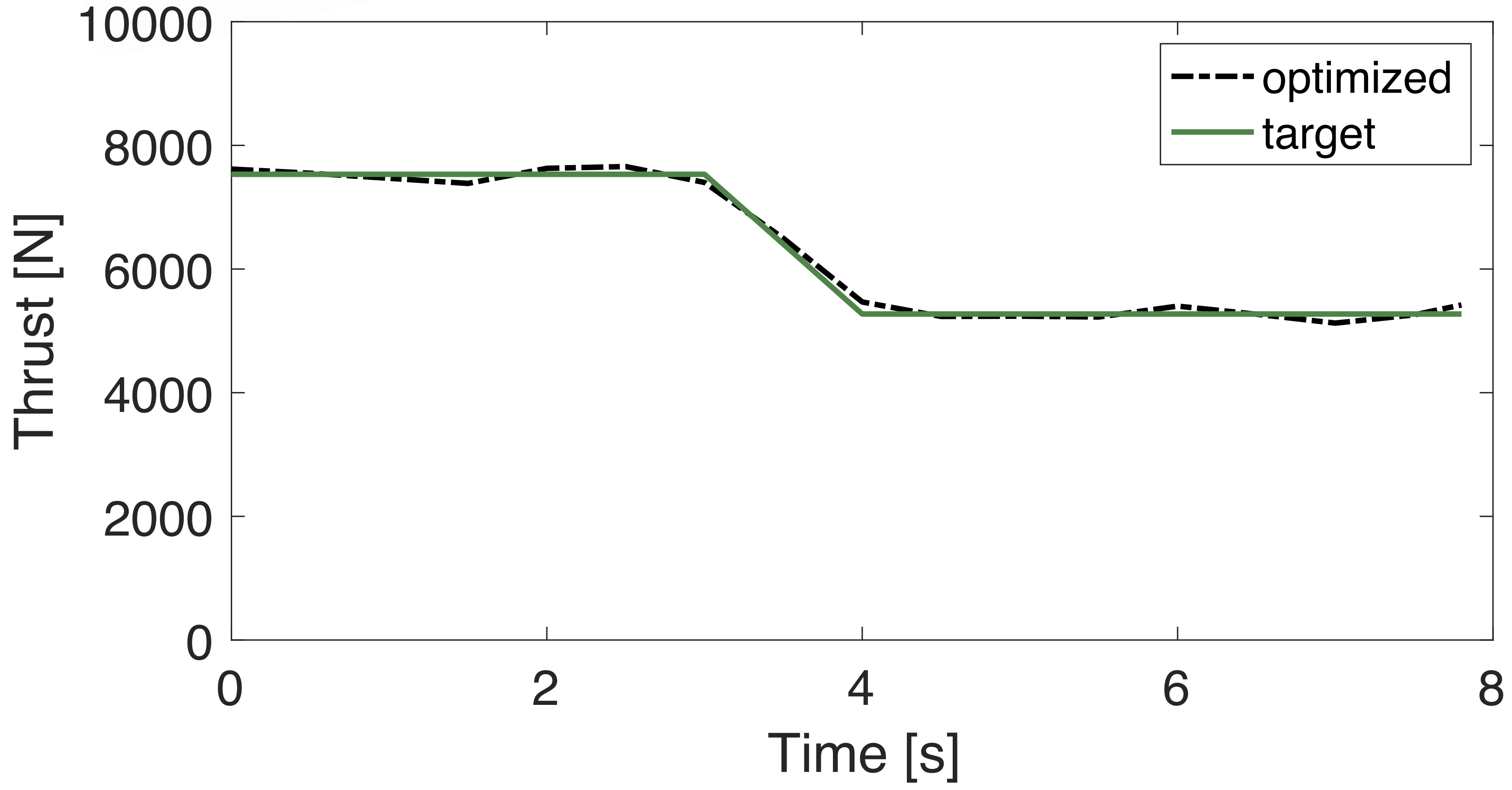
Solid Rocket Fuel Design



$$\min_{\boldsymbol{w}} \quad \sum_t \left(th(\boldsymbol{w}) - th_{target}\right)^2$$

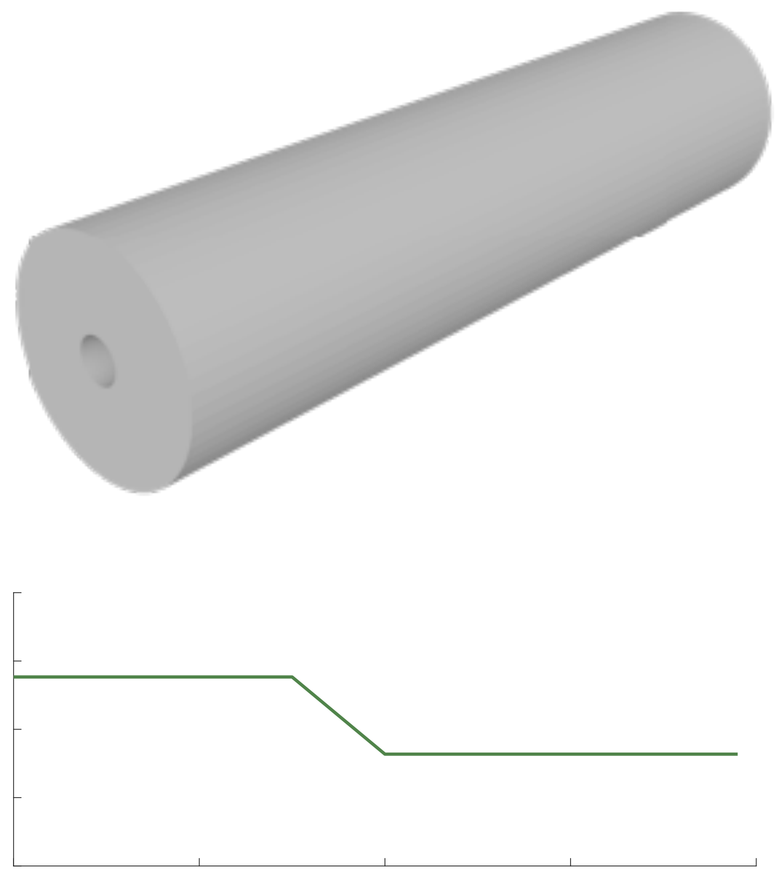
$$\text{s.t.} \quad r_b(\boldsymbol{w})^i > r_{in}$$



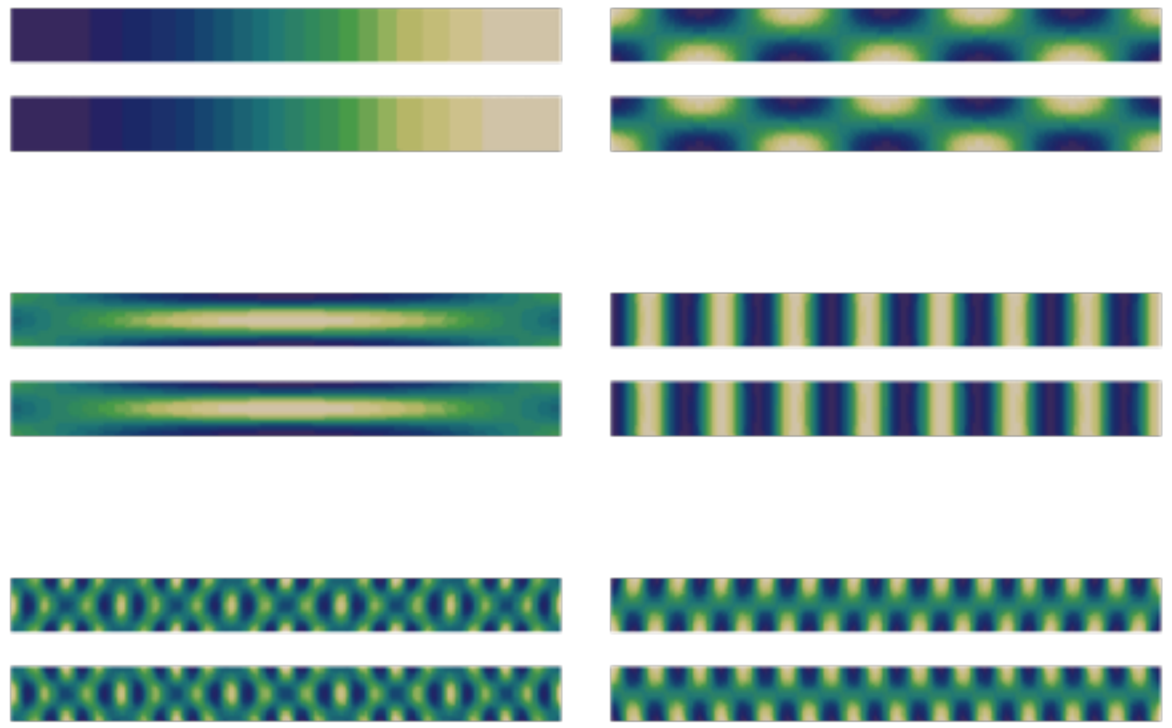


Sliding Basis Optimization

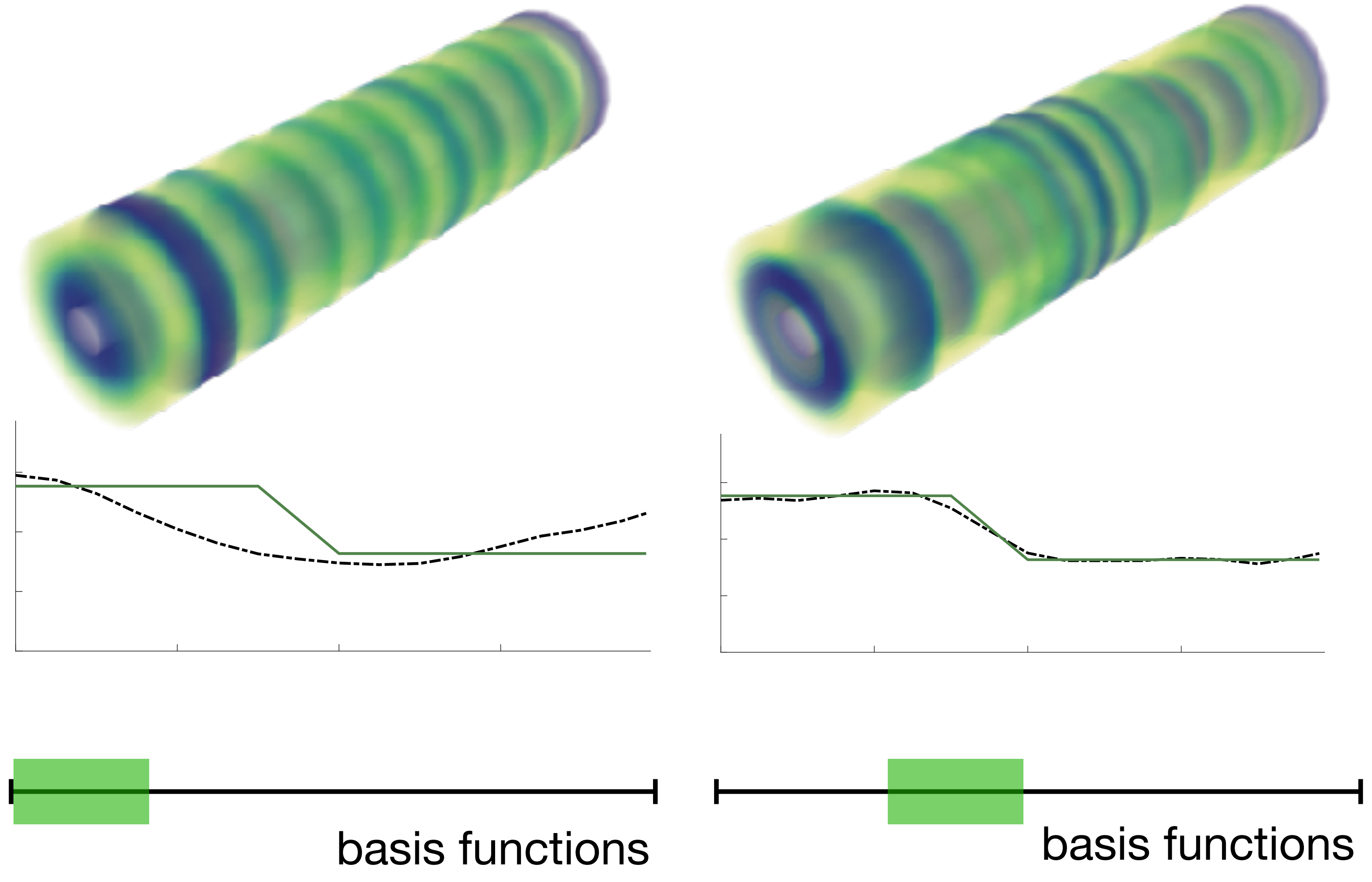
Inputs



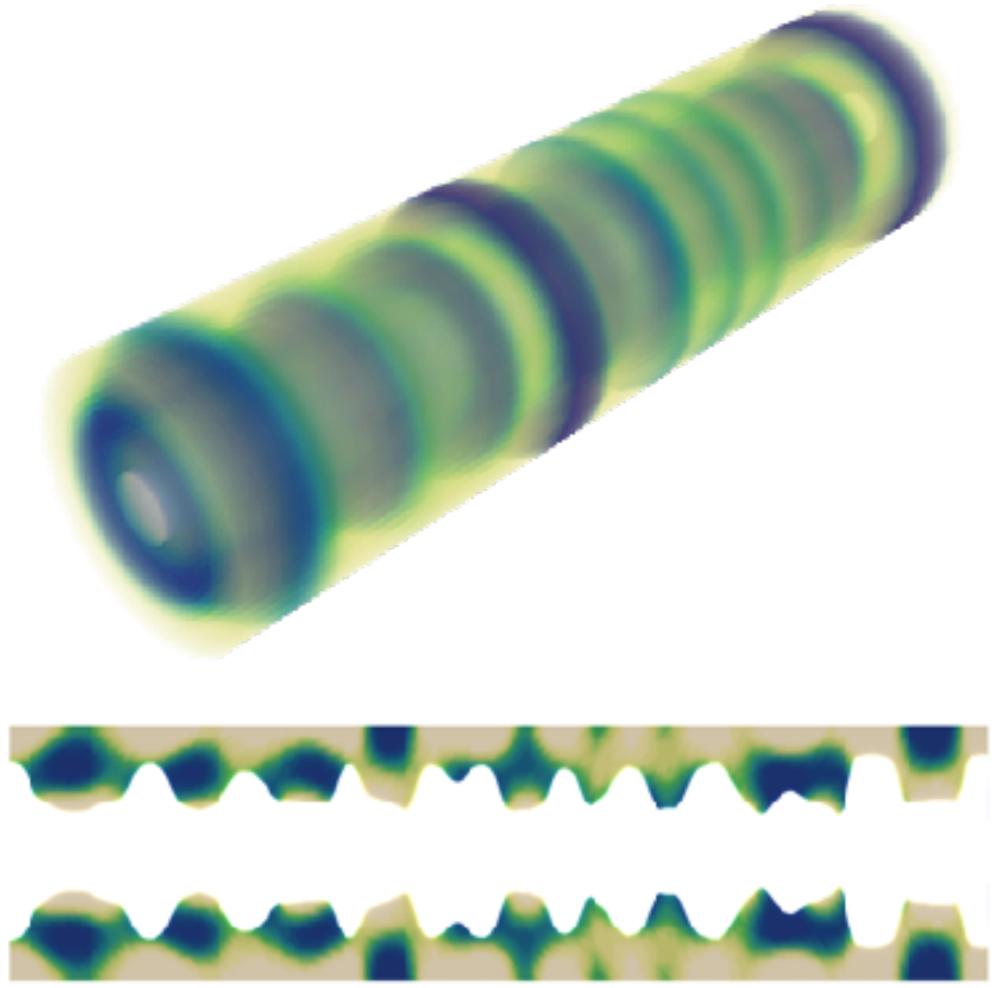
Compute Basis Functions



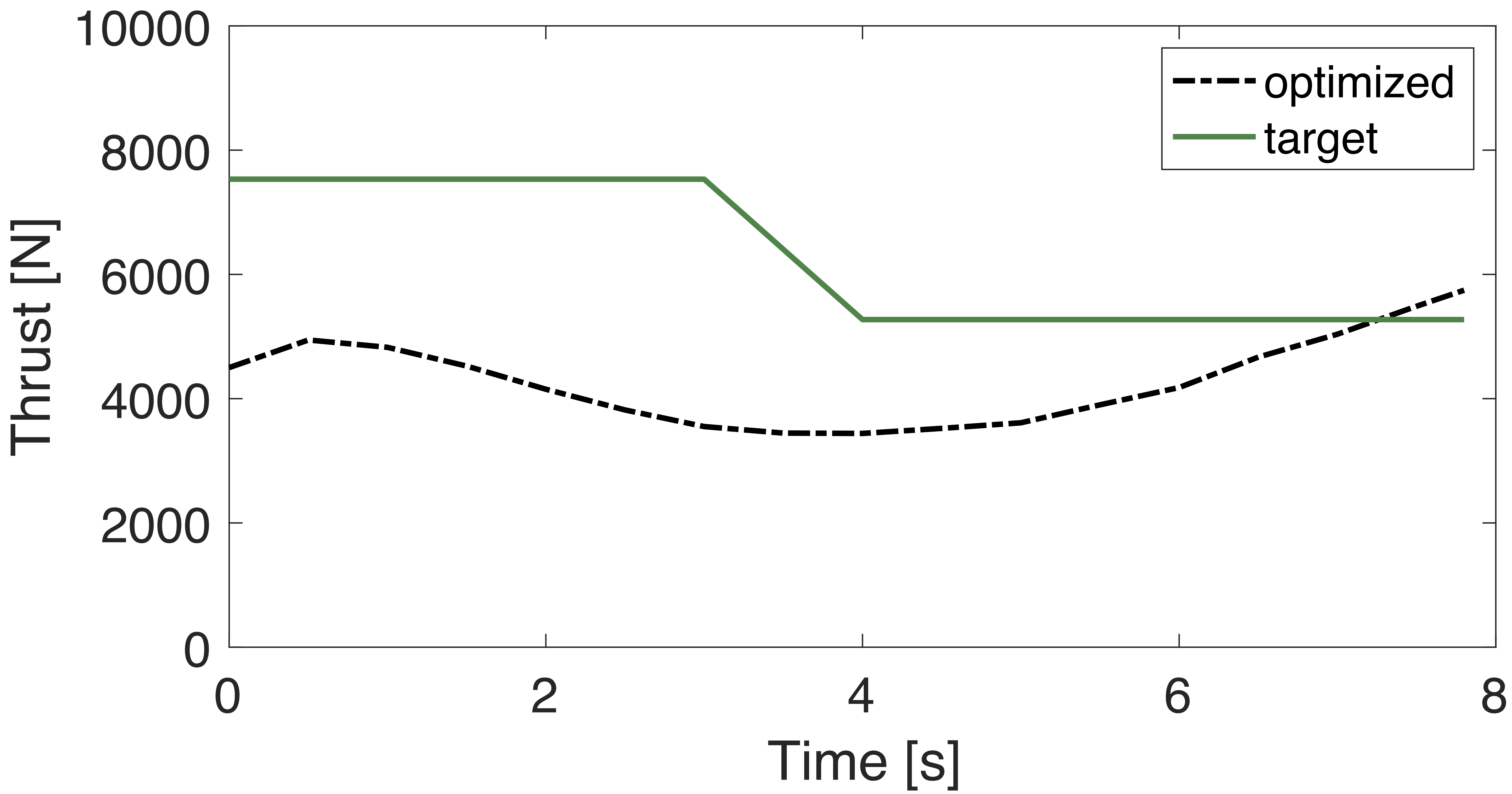
Optimization



Output



Progression of thrust profile match through the sliding basis optimization



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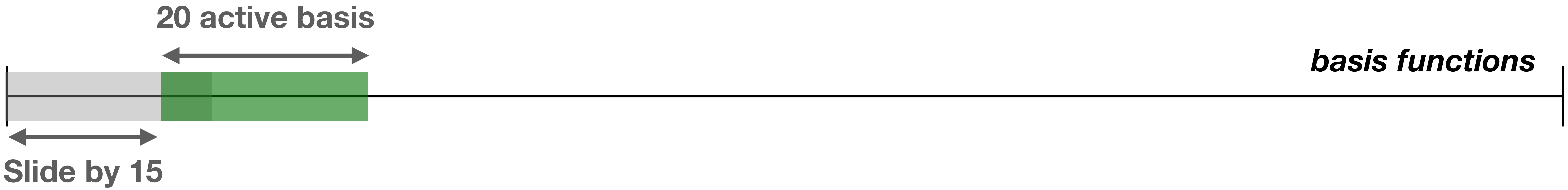

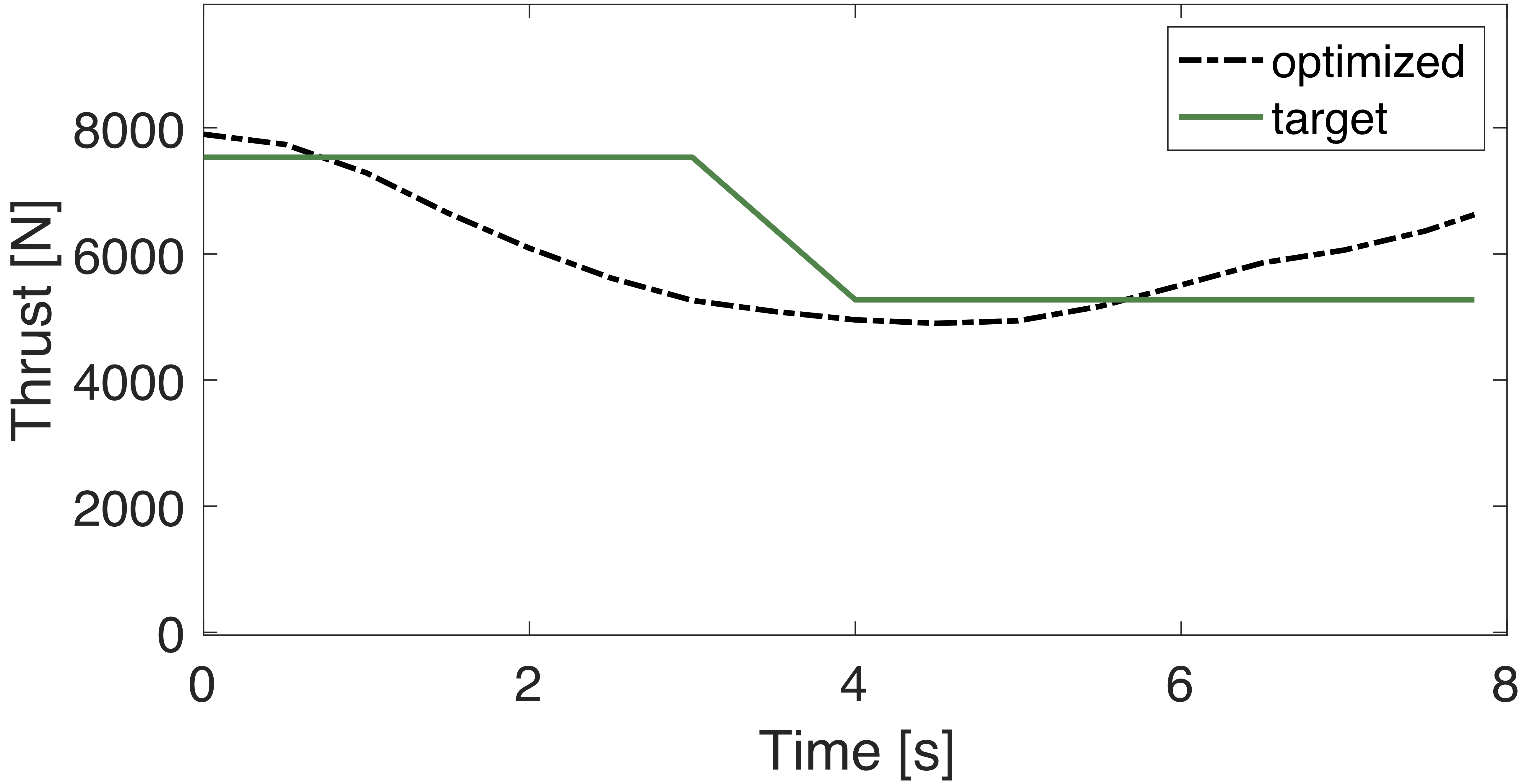

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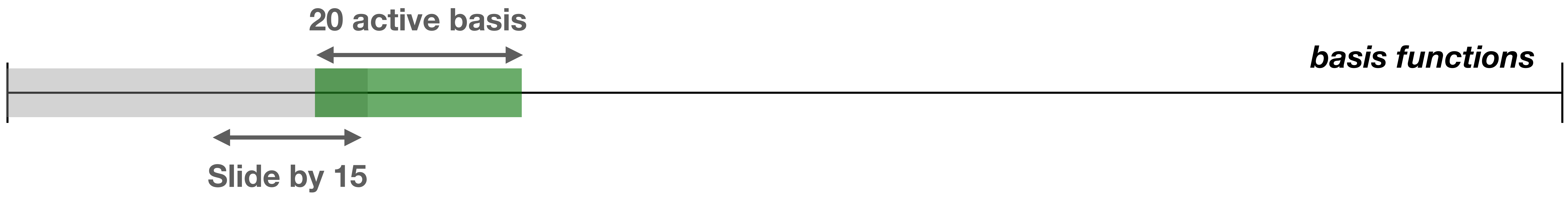
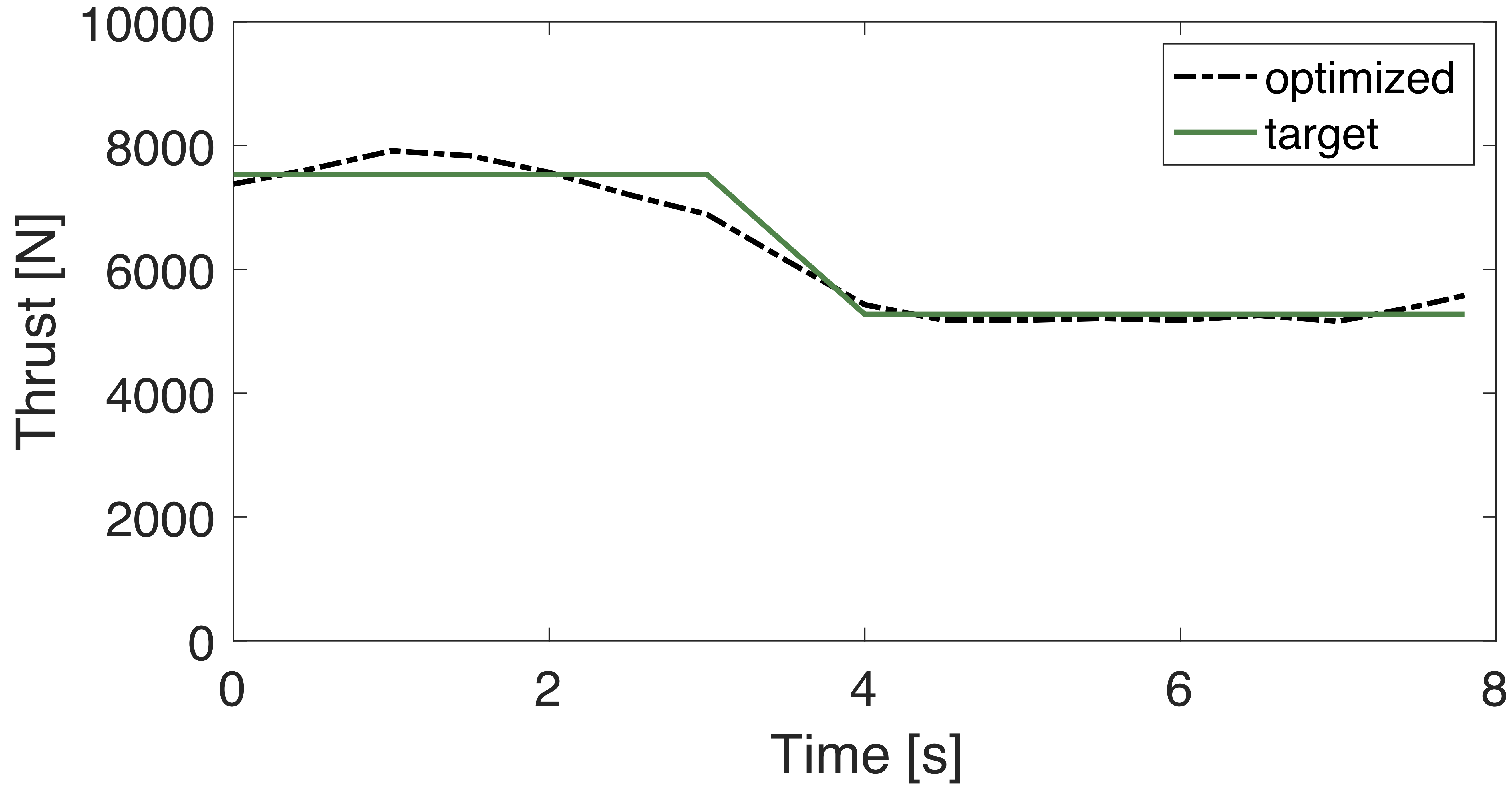


20 active basis

basis functions

Slide by 15



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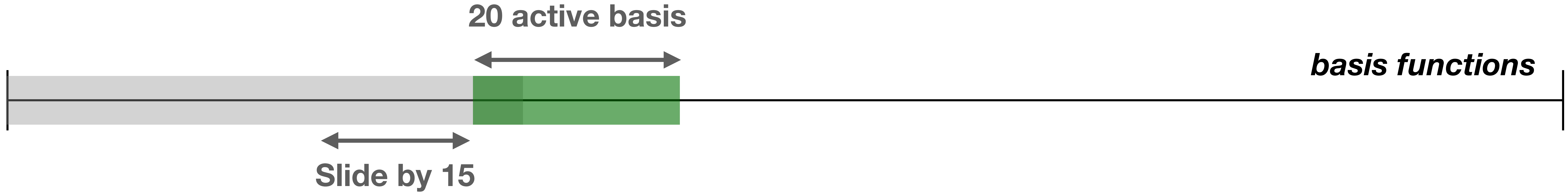

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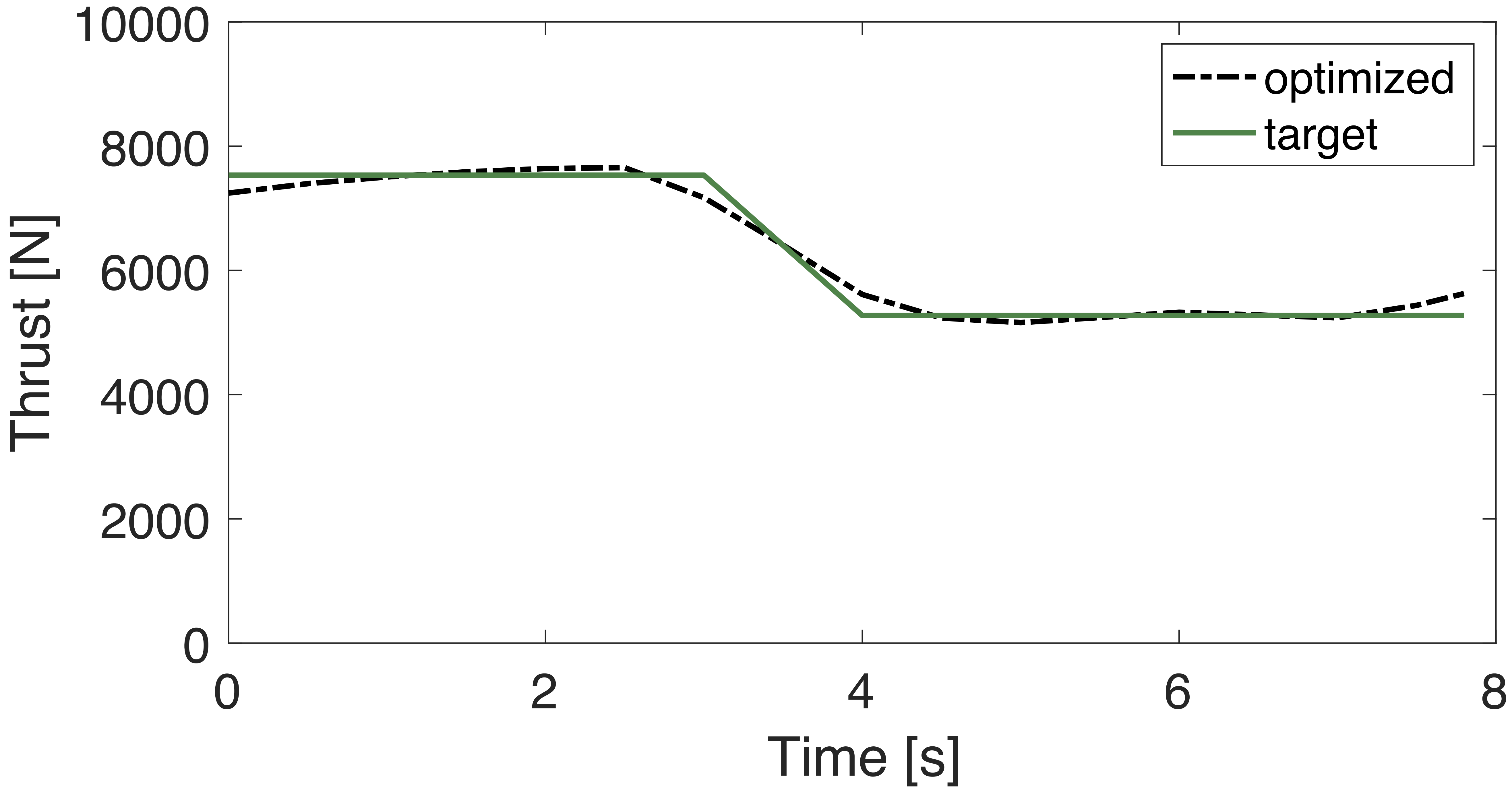

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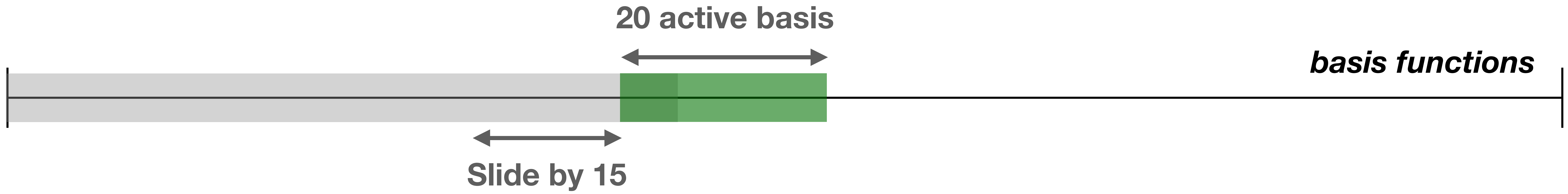

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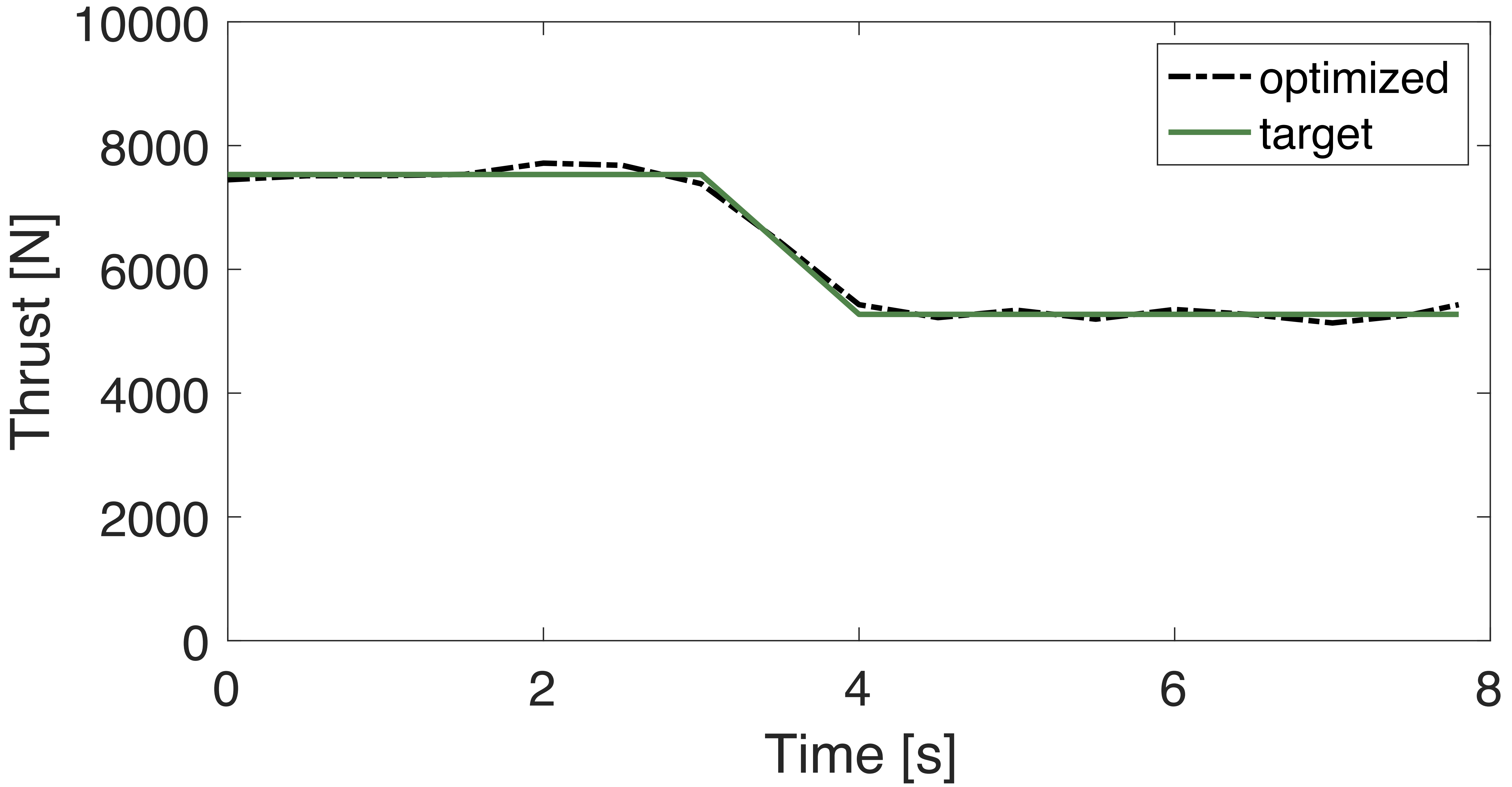

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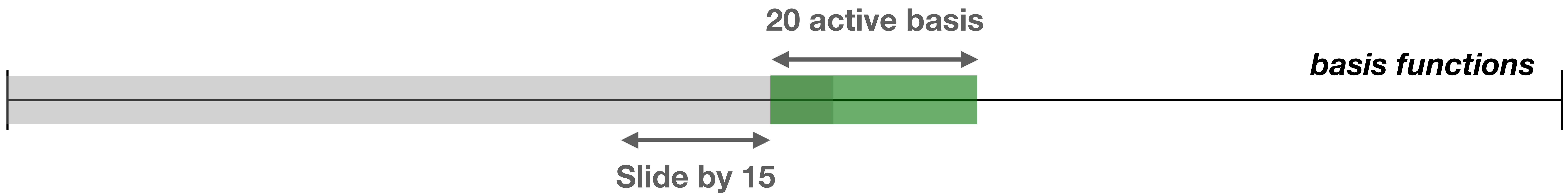
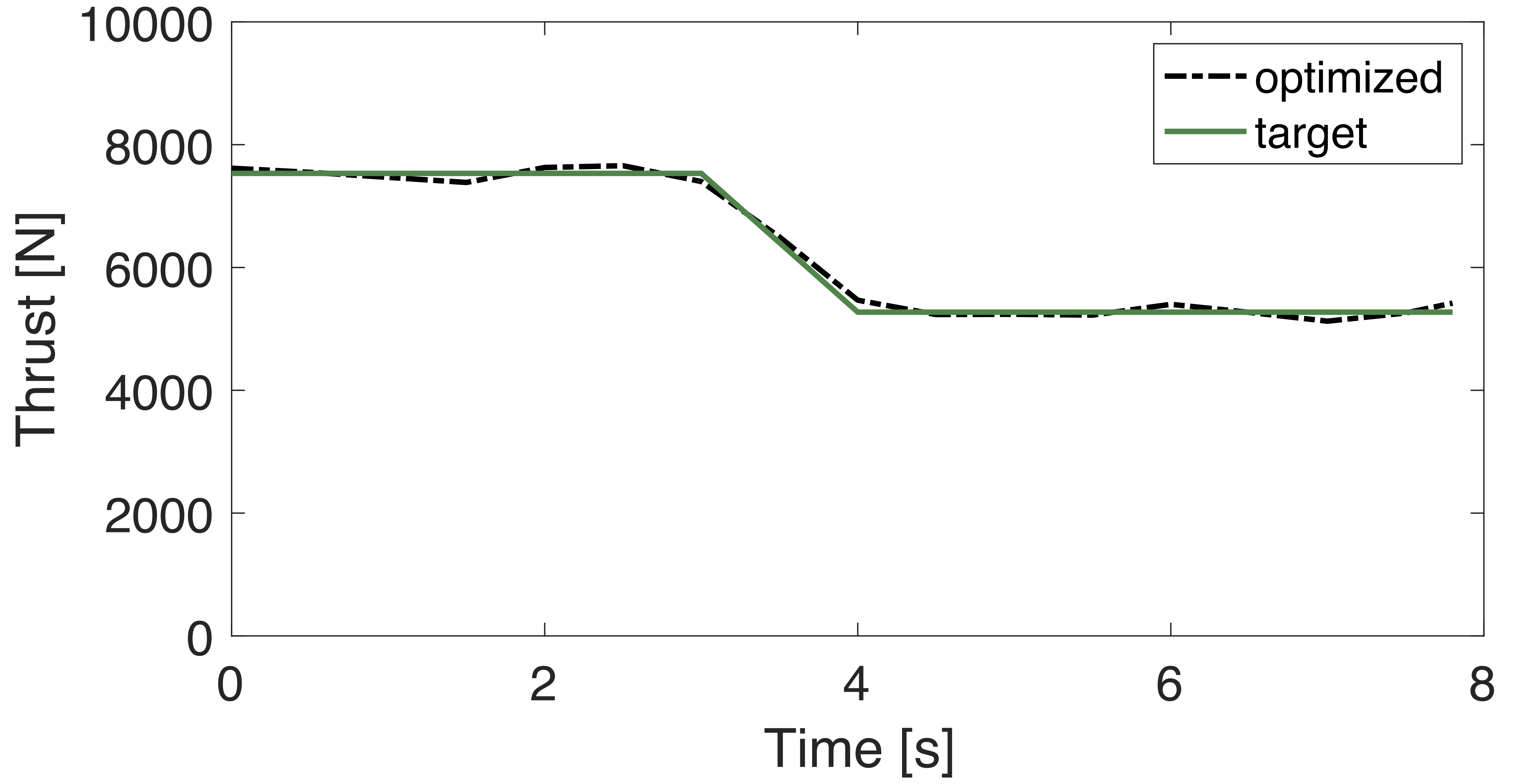


20 active basis

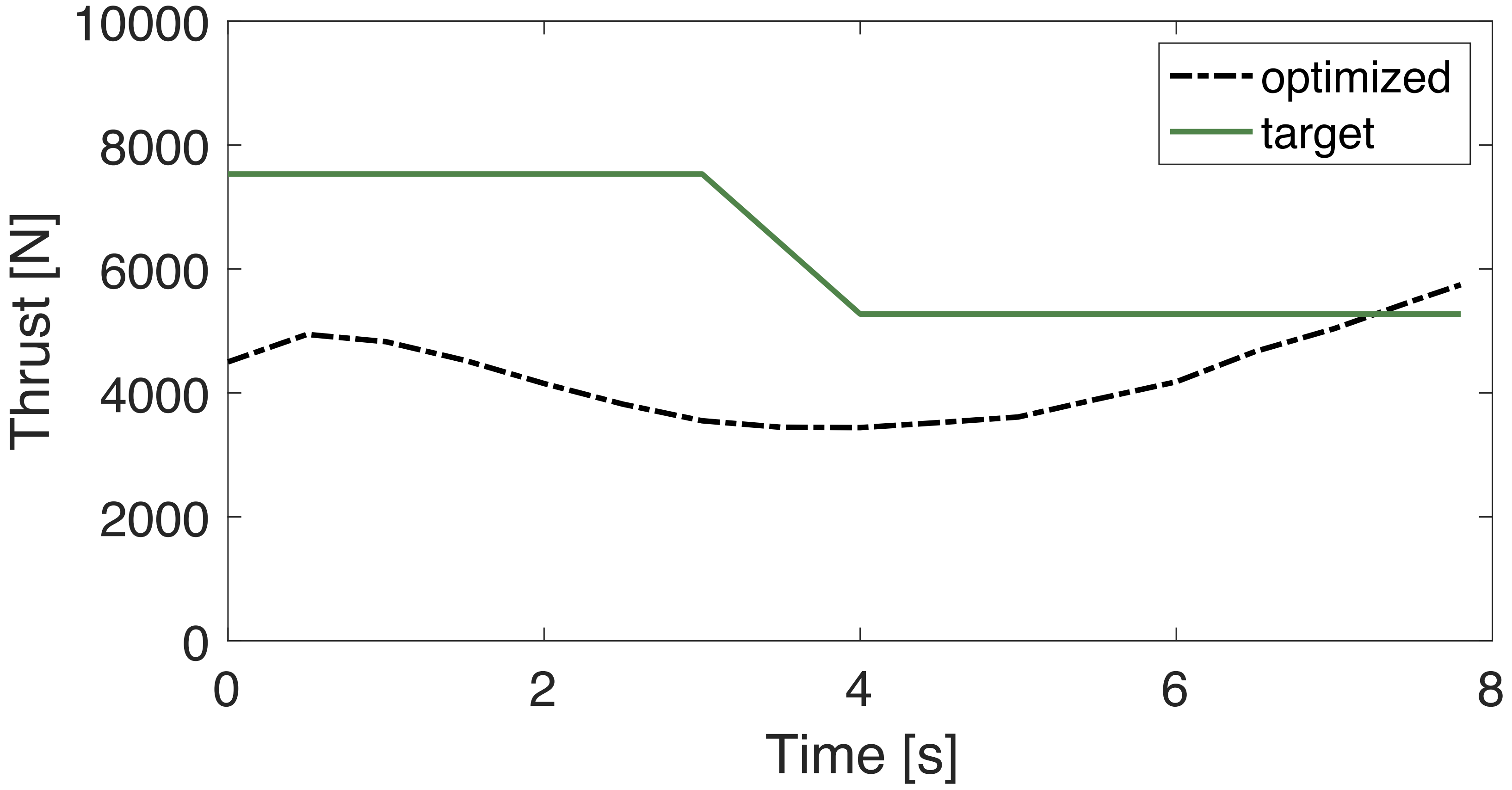
basis functions

Slide by 15

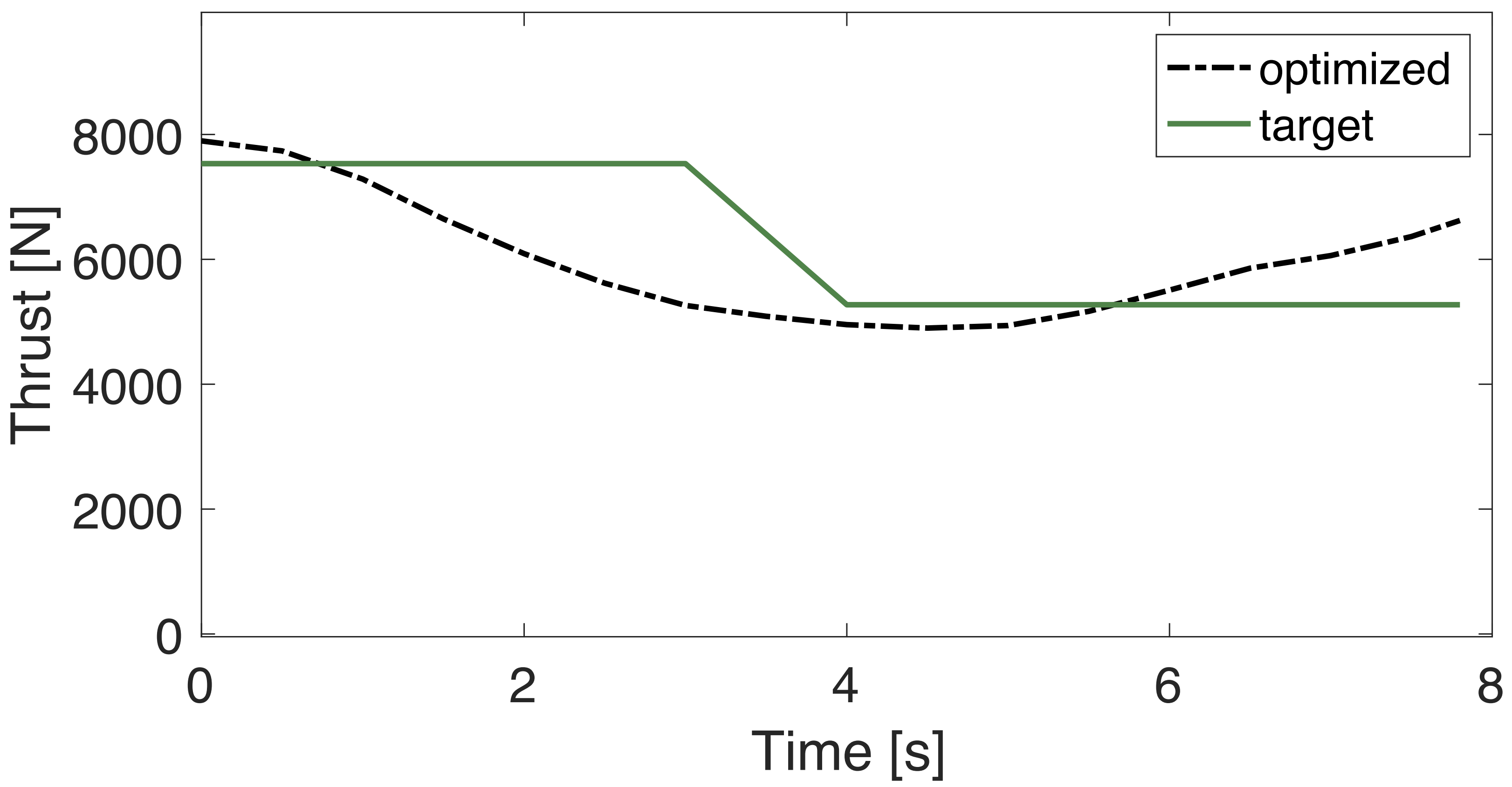


Progression of thrust profile match through the sliding basis optimization

First 20 basis



Slide by 15



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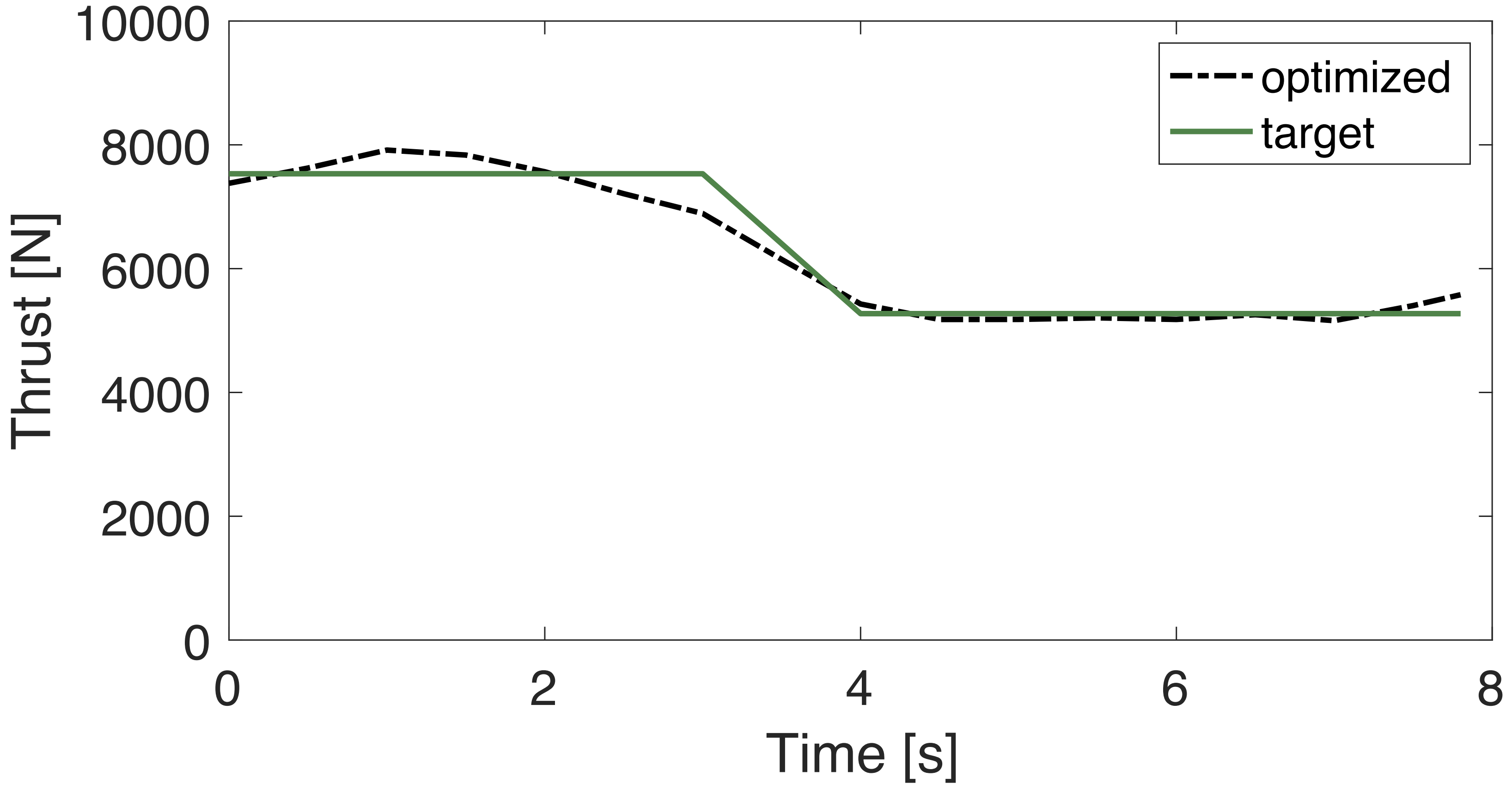

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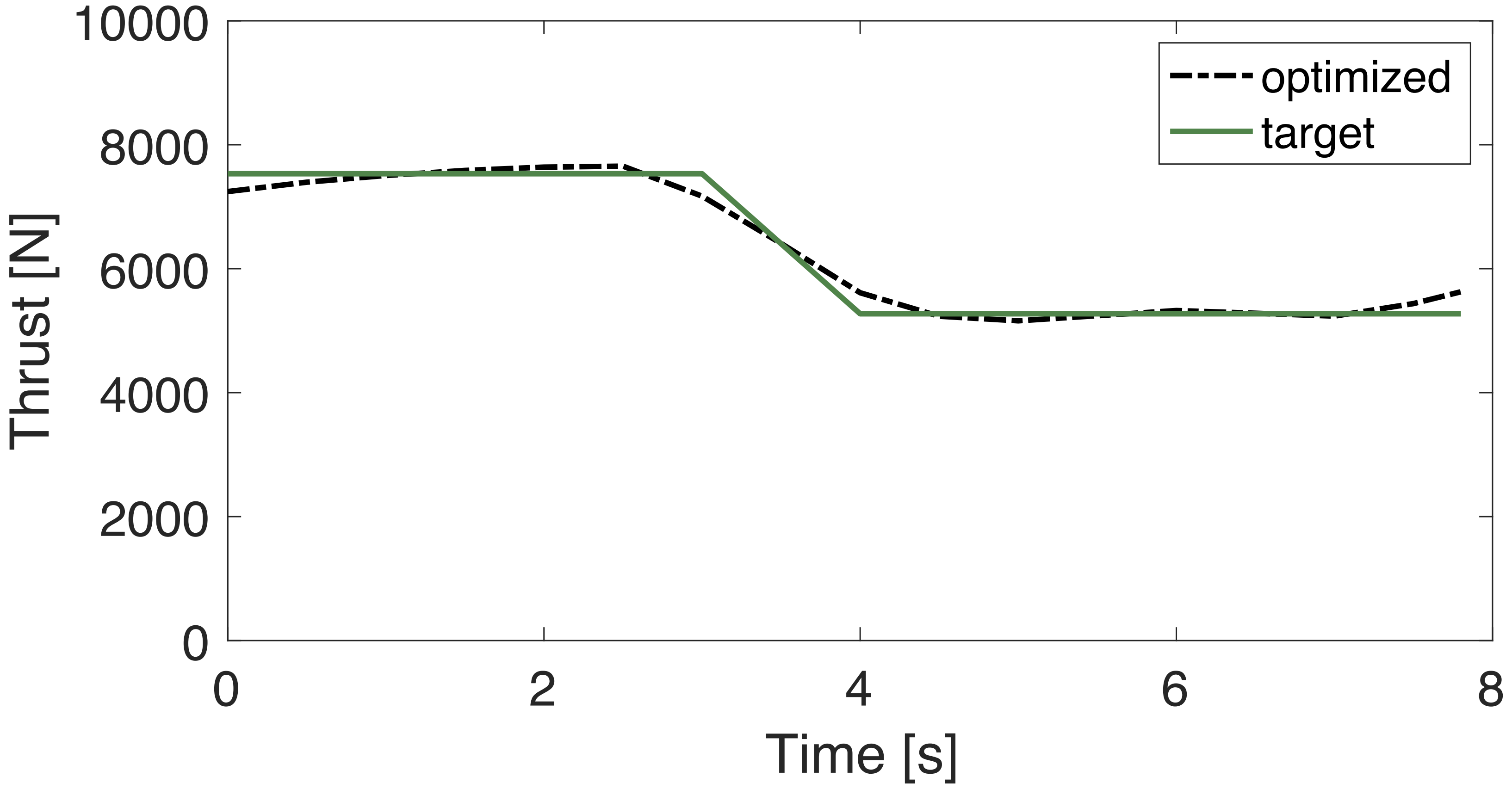

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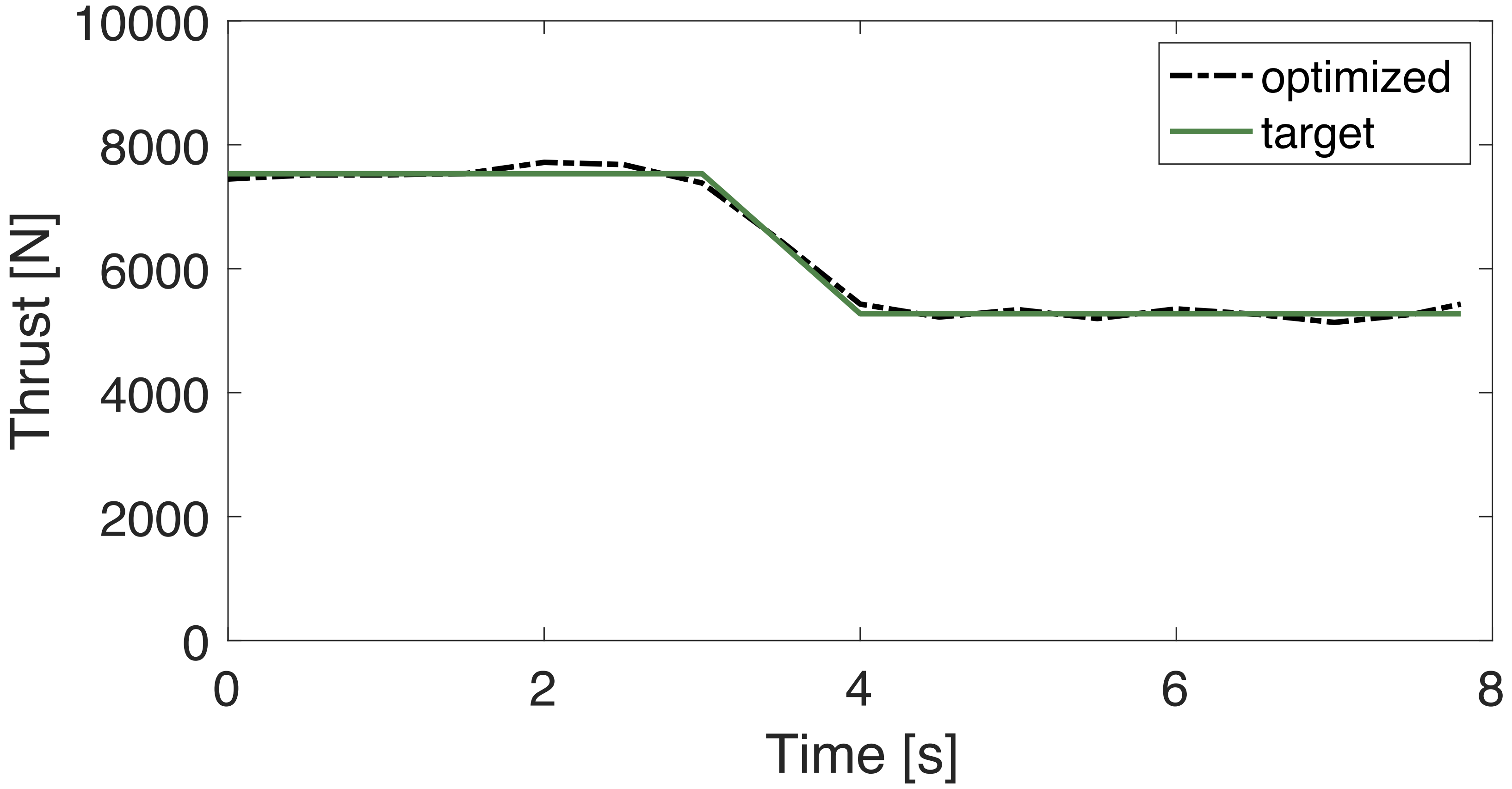
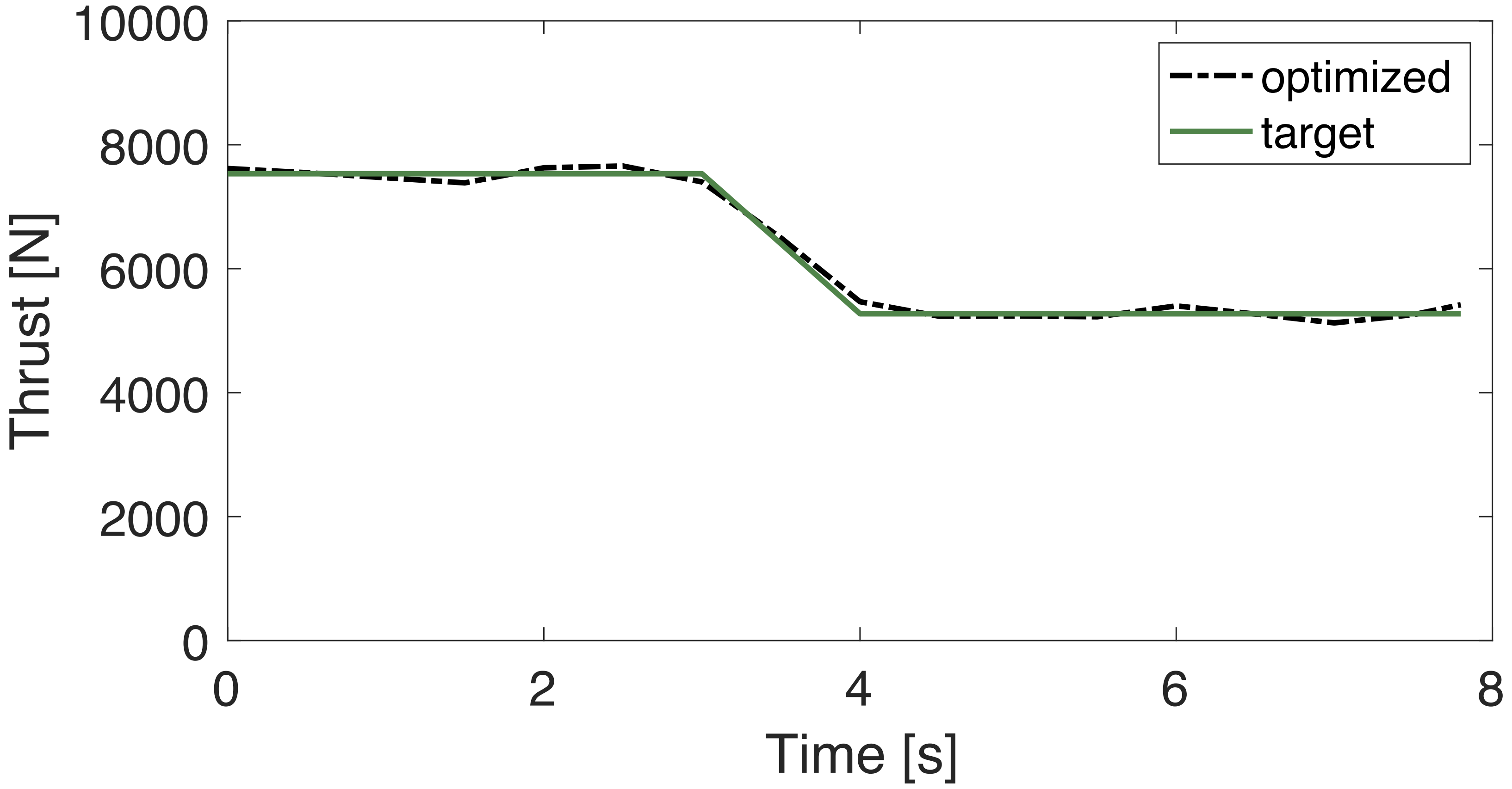
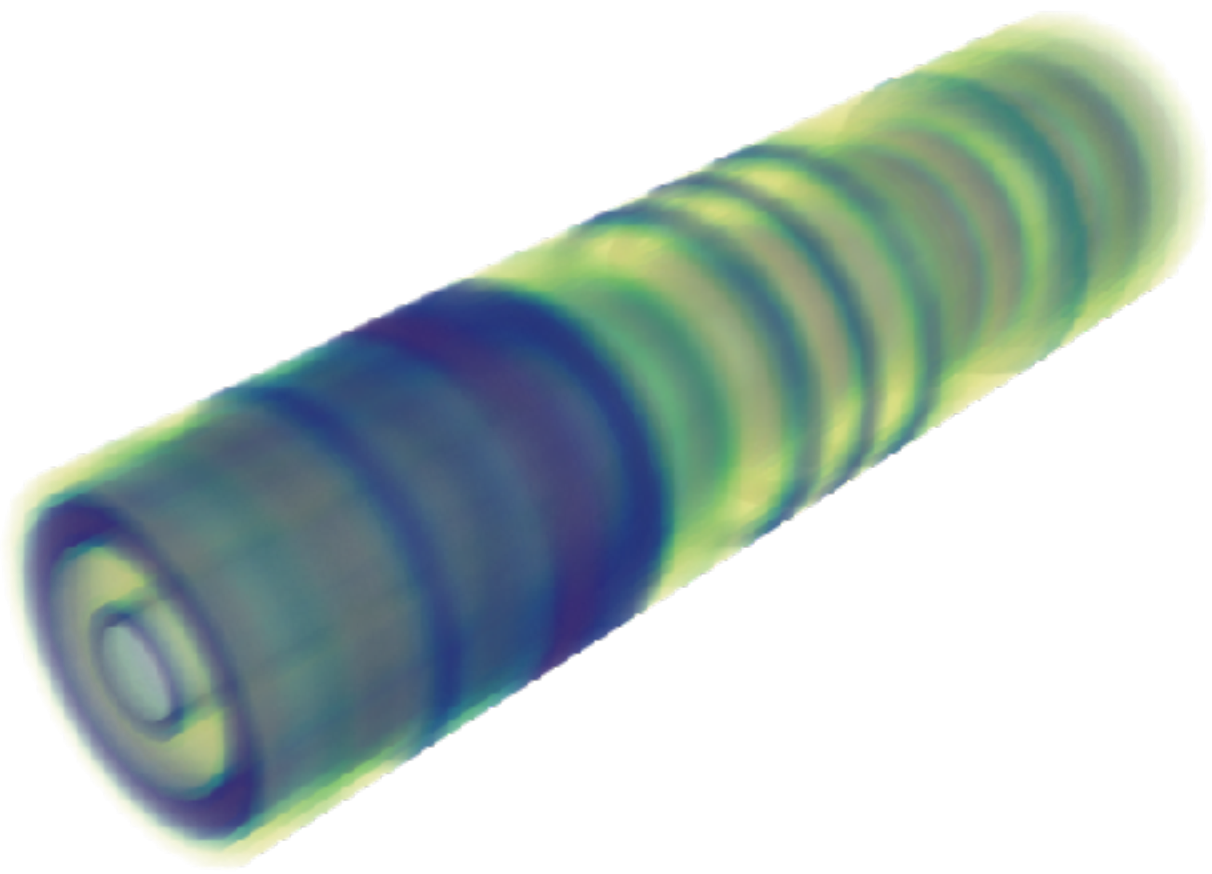
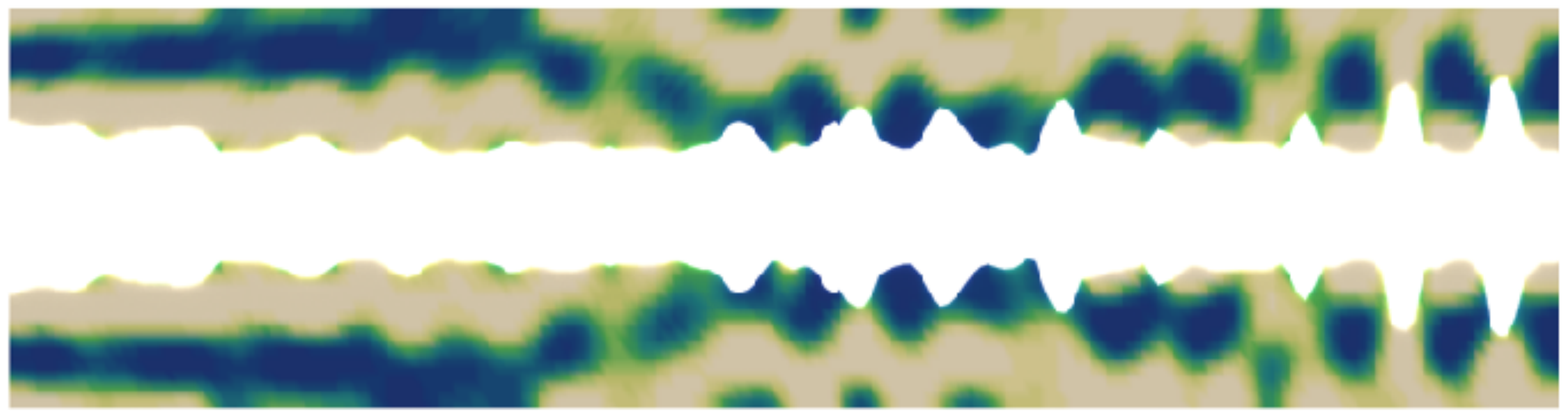


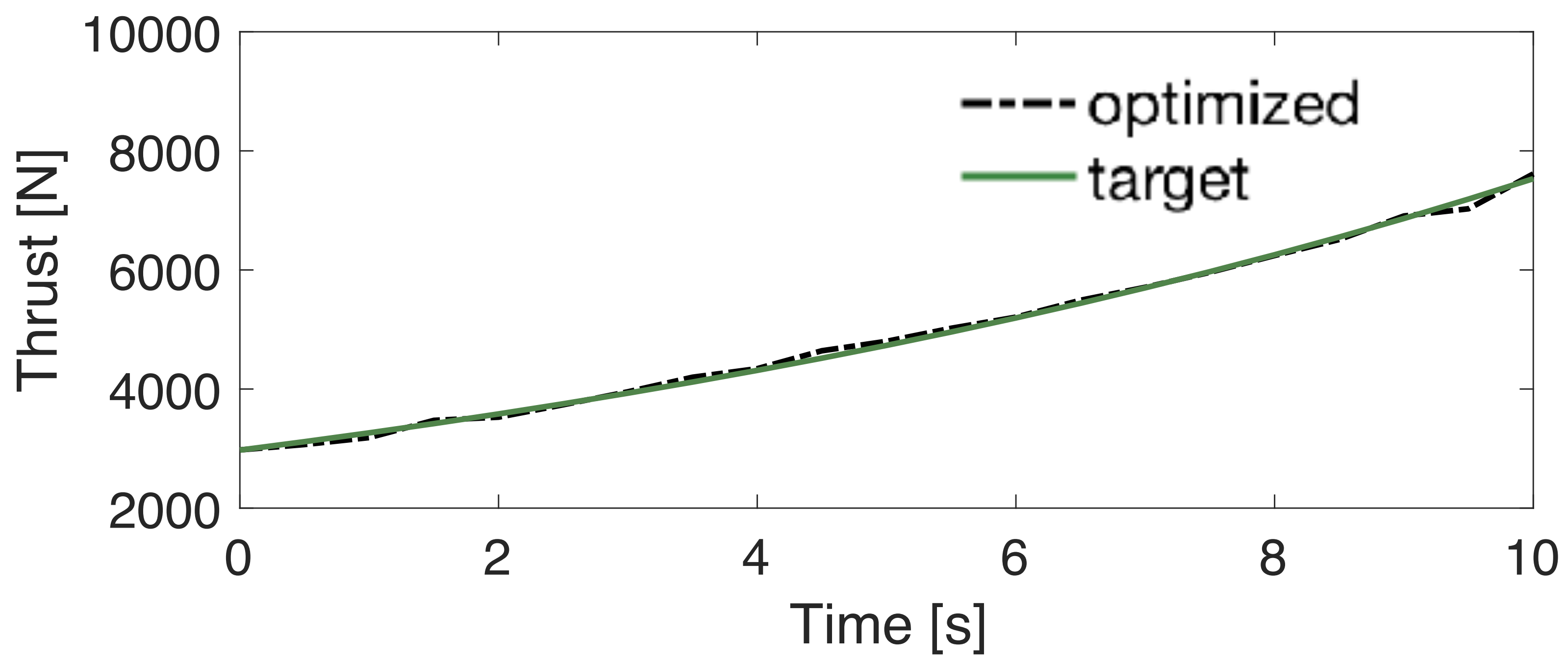
Slide by 15

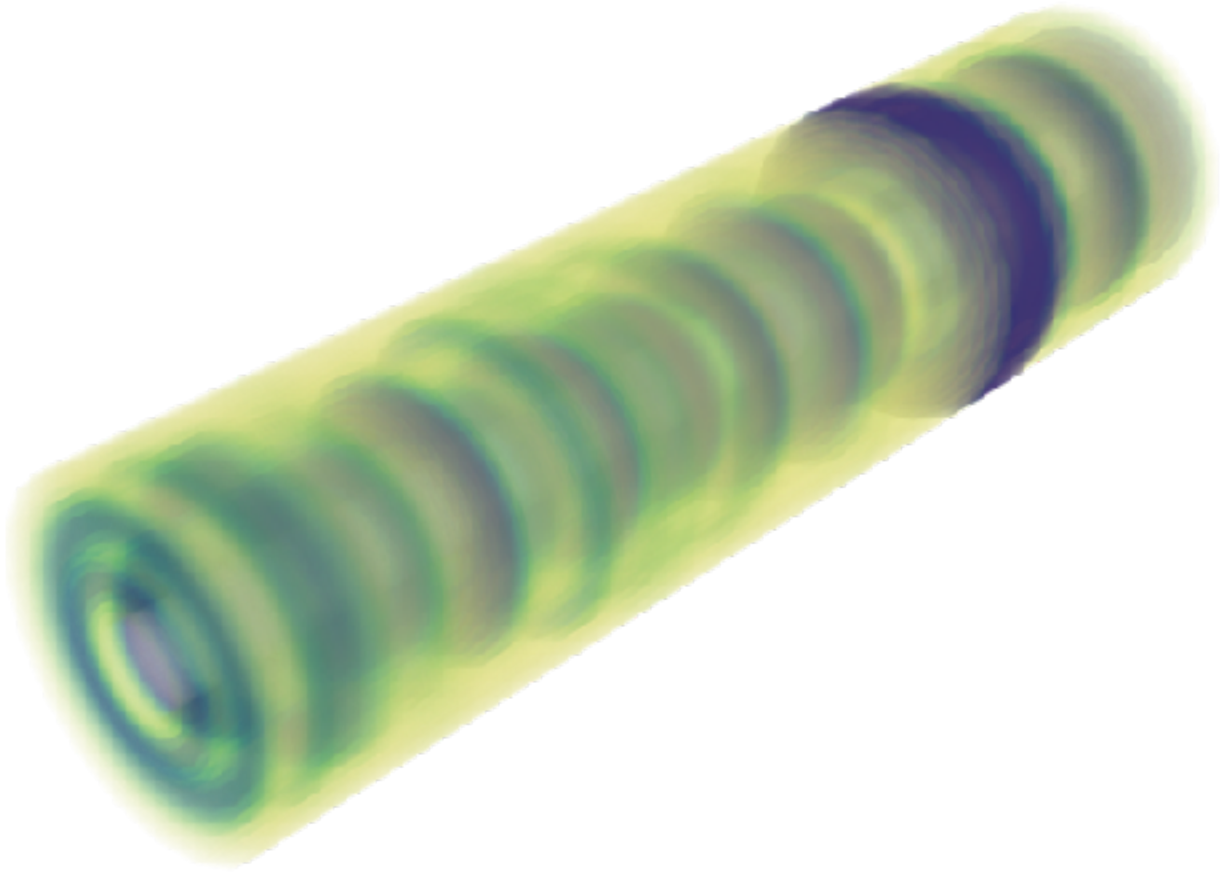


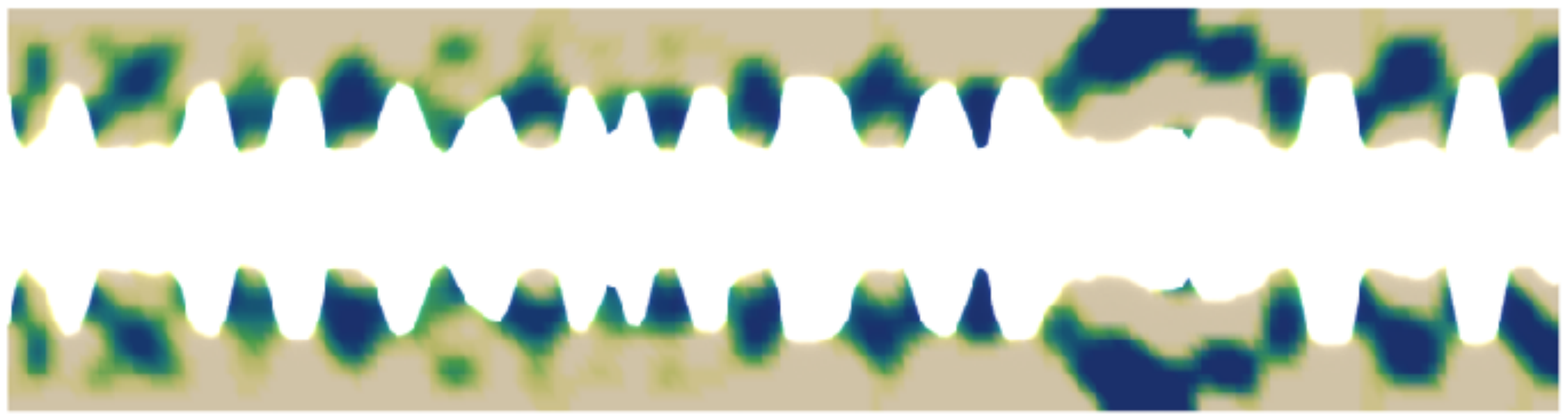
Optimized Solid Rocket Fuel Designs

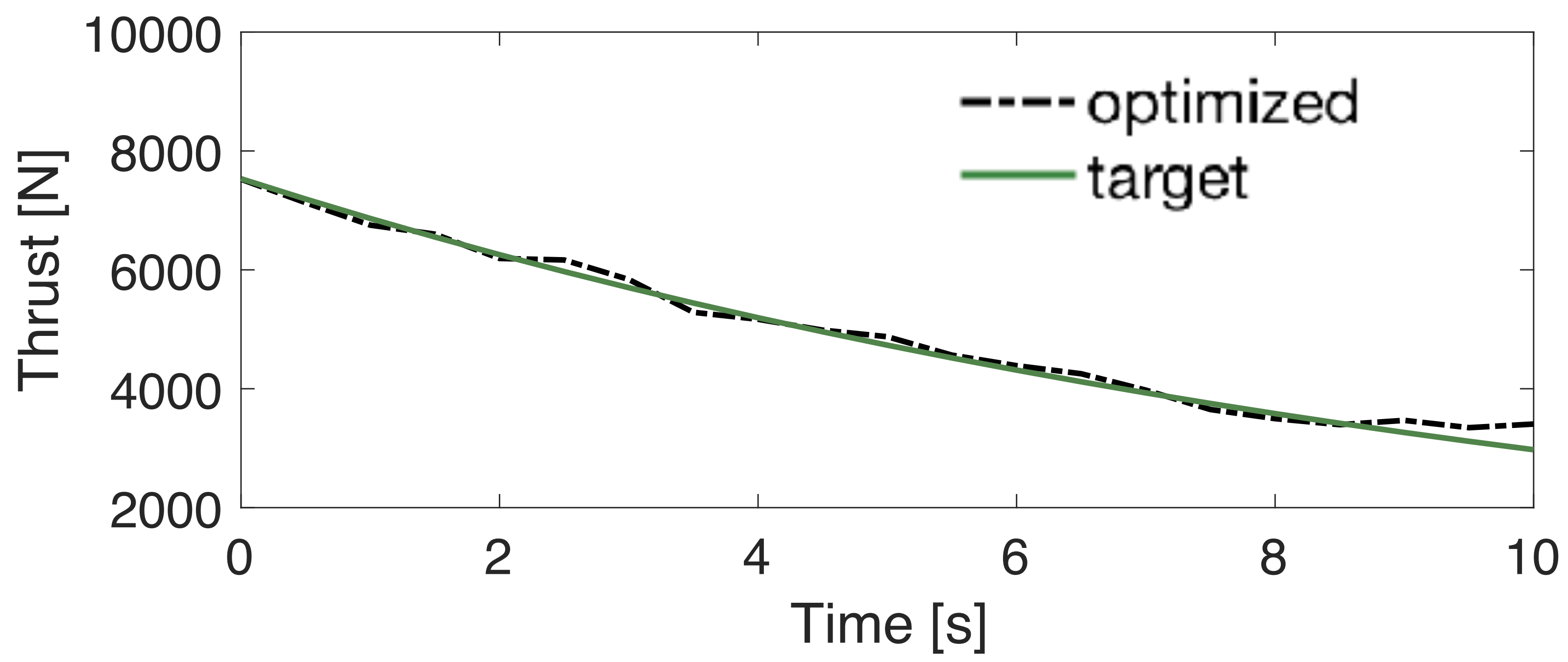












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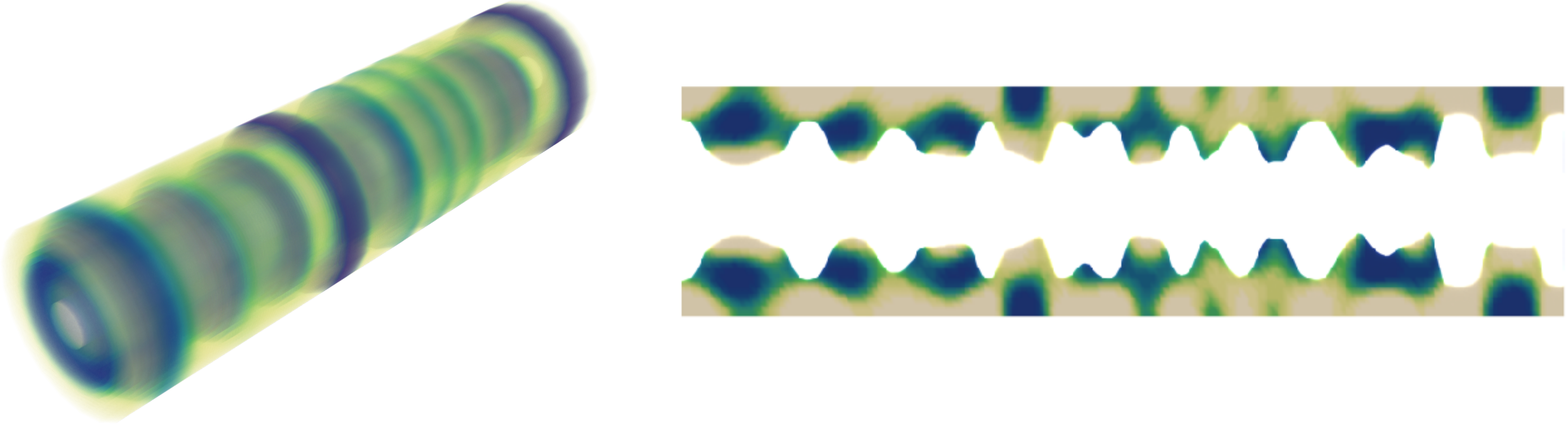
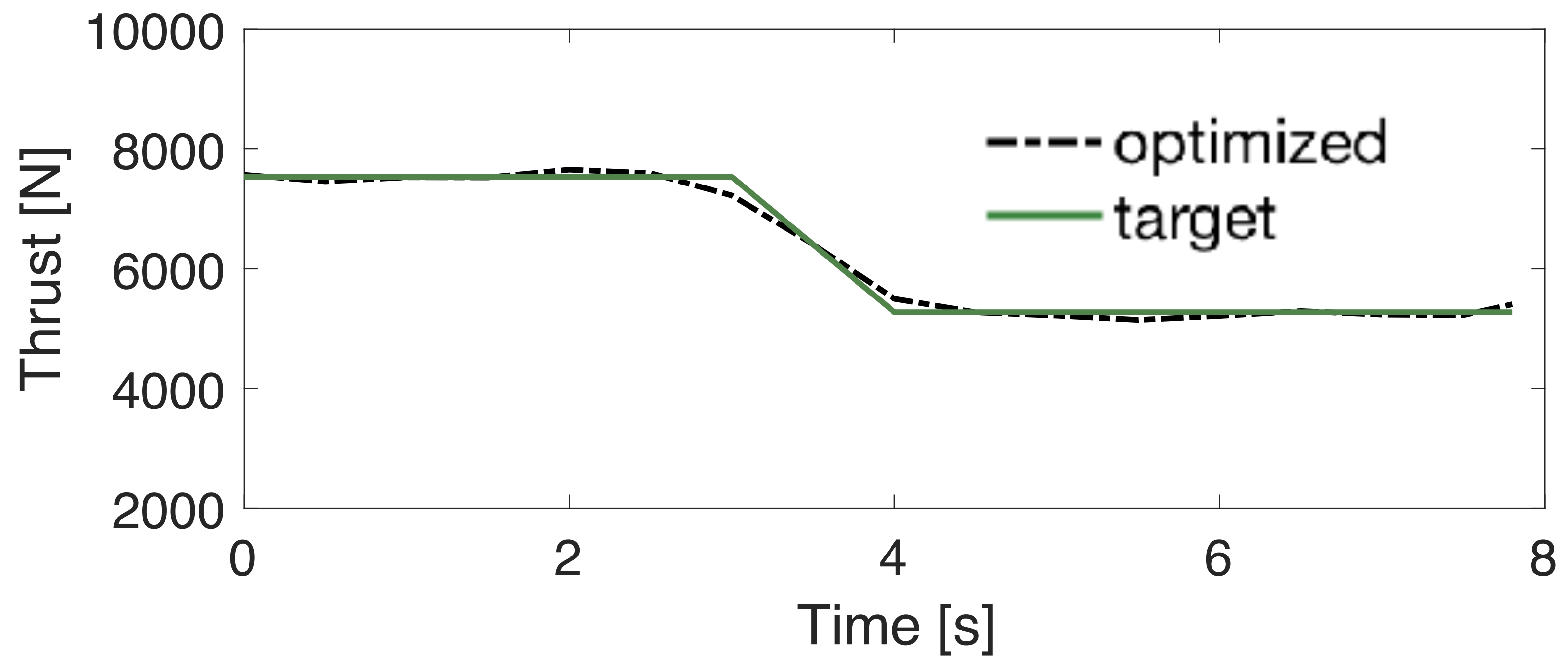
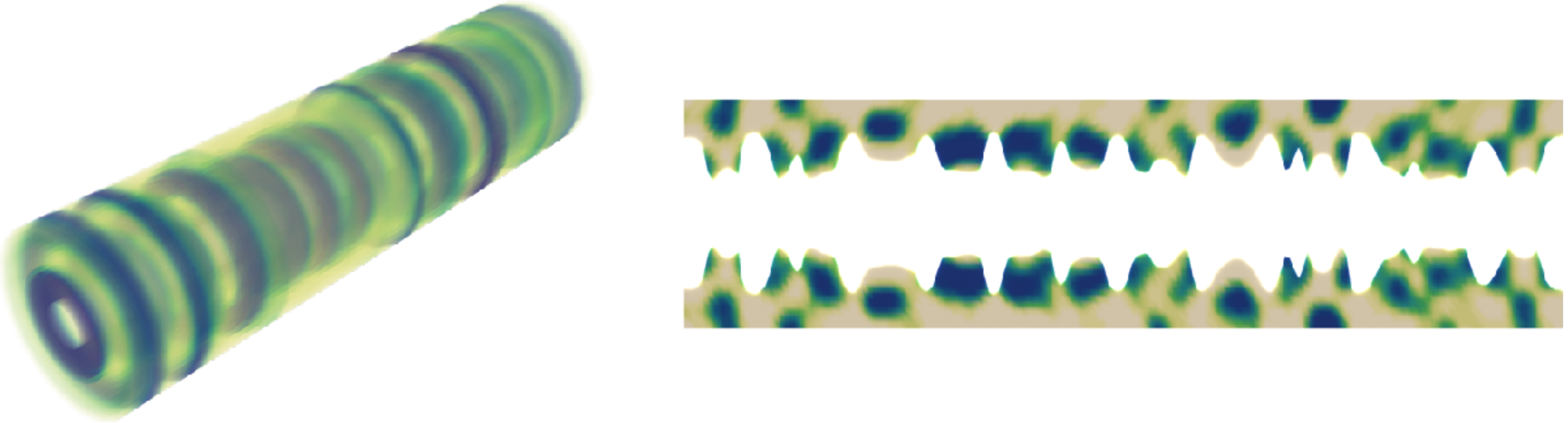
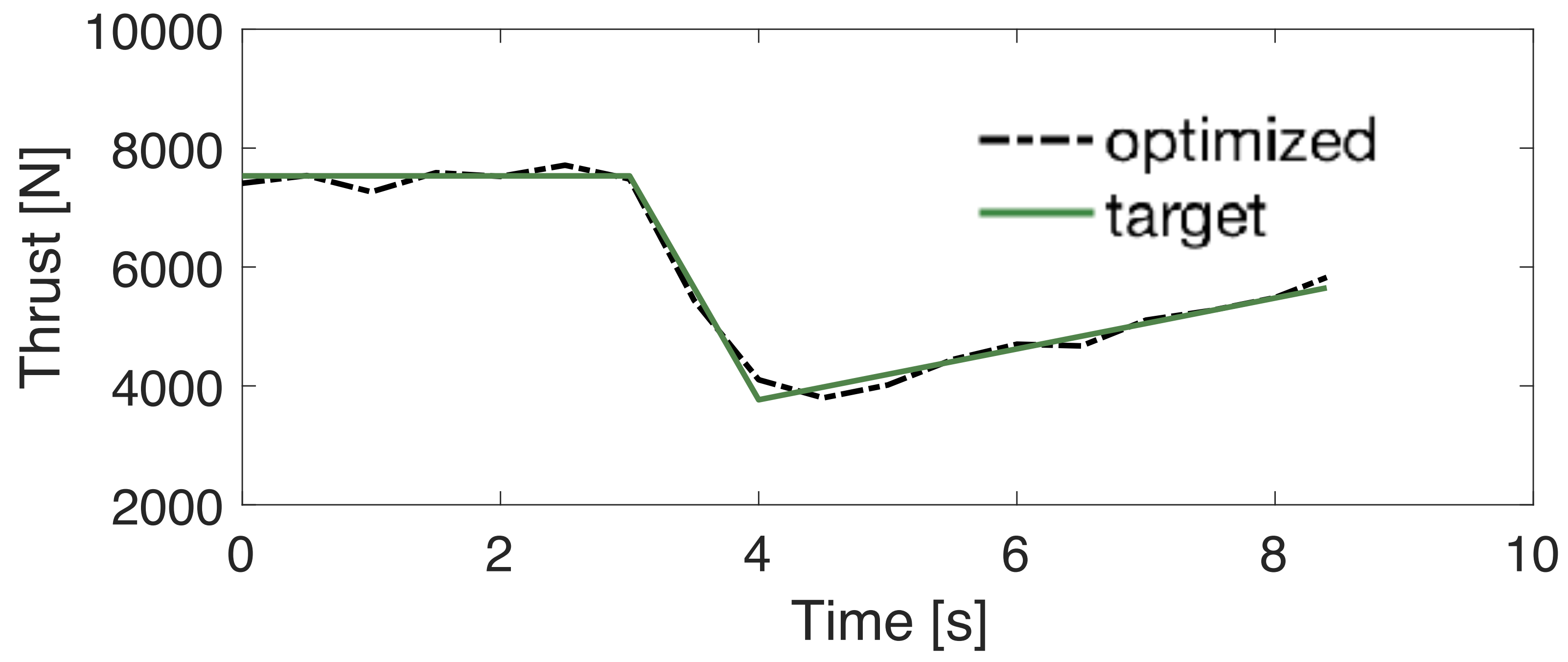


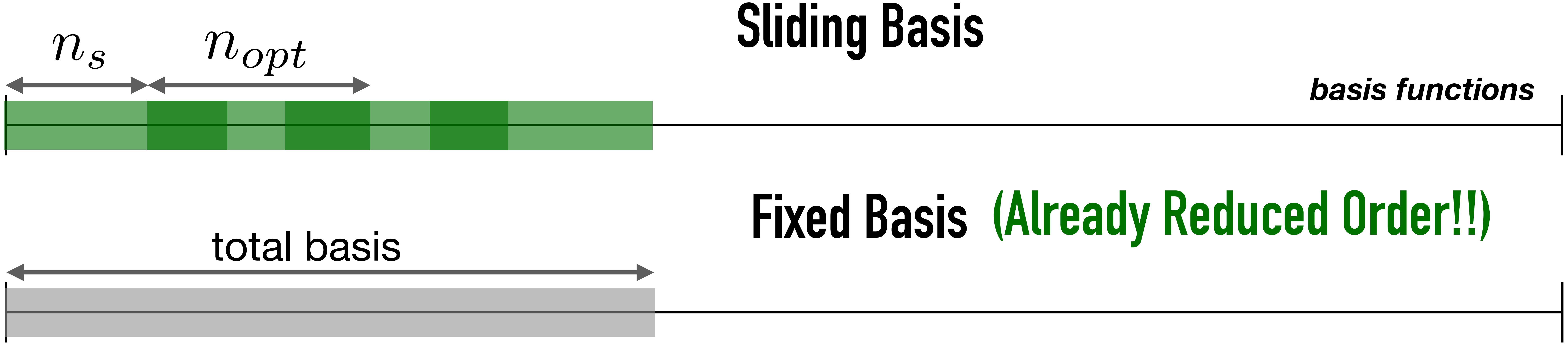






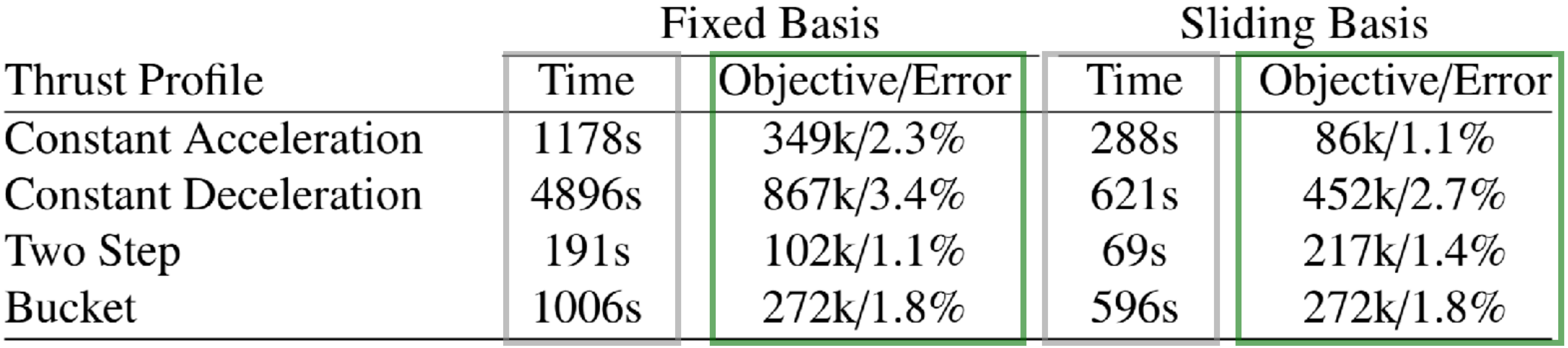
Graded material fields!

Performance



Sliding Basis

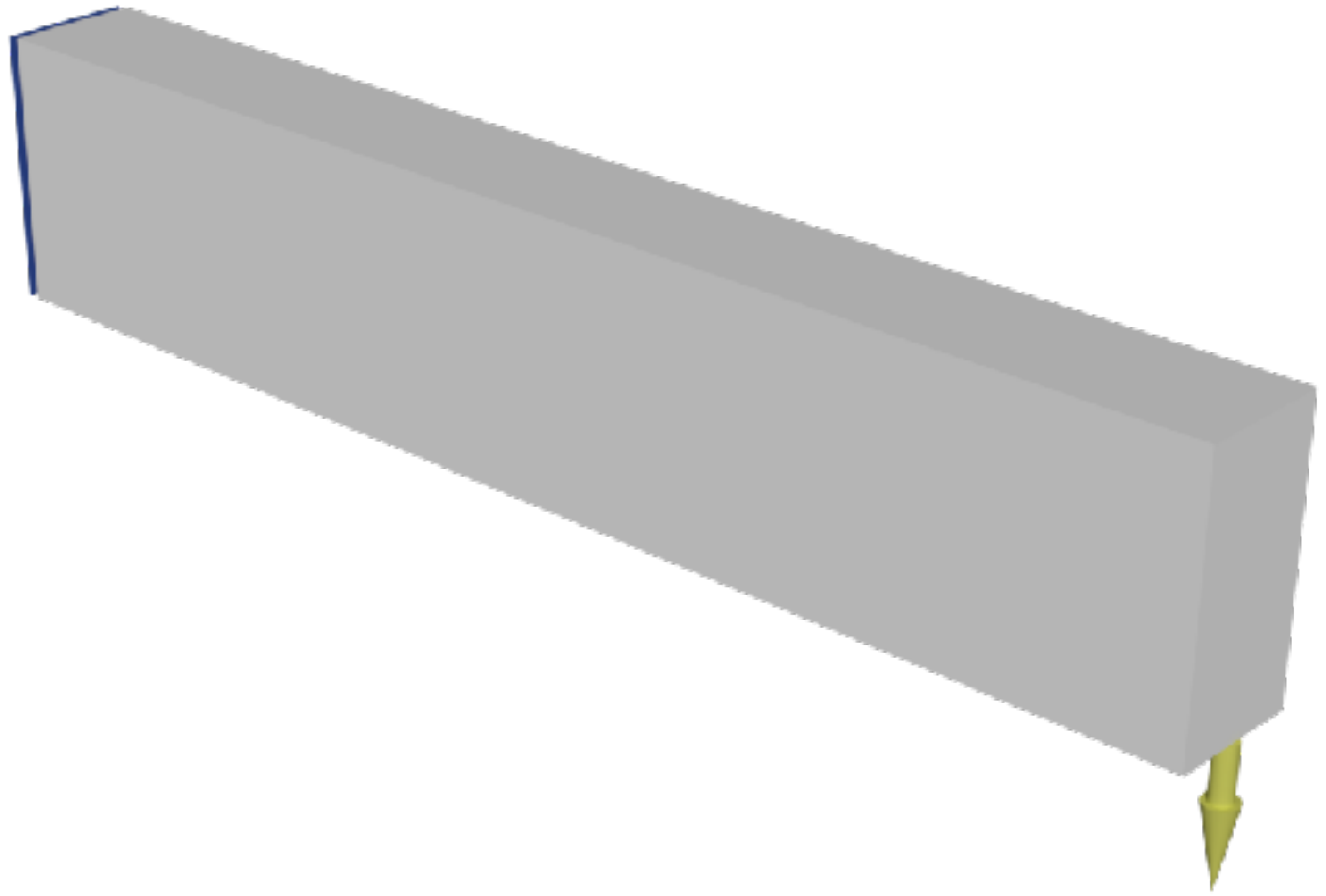
Fixed Basis (Already Reduced Order!!)



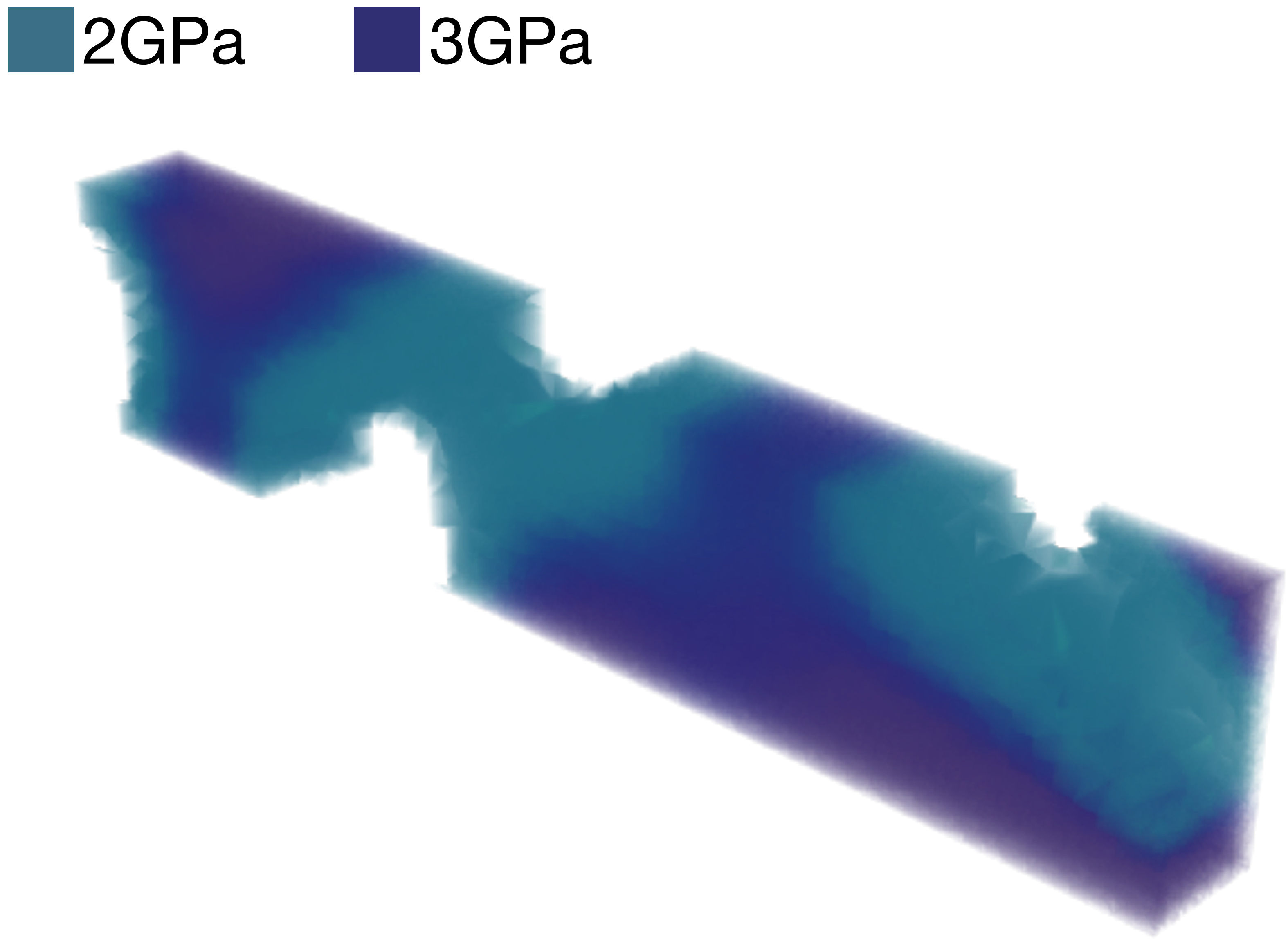
	Fixed Basis		Sliding Basis	
Thrust Profile	Time	Objective/Error	Time	Objective/Error
Constant Acceleration	1178s	349k/2.3%	288s	86k/1.1%
Constant Deceleration	4896s	867k/3.4%	621s	452k/2.7%
Two Step	191s	102k/1.1%	69s	217k/1.4%
Bucket	1006s	272k/1.8%	596s	272k/1.8%

Upto 8x speed up **Comparable objective values**

Multi-material Topology Optimization Through Sliding Optimization Steps



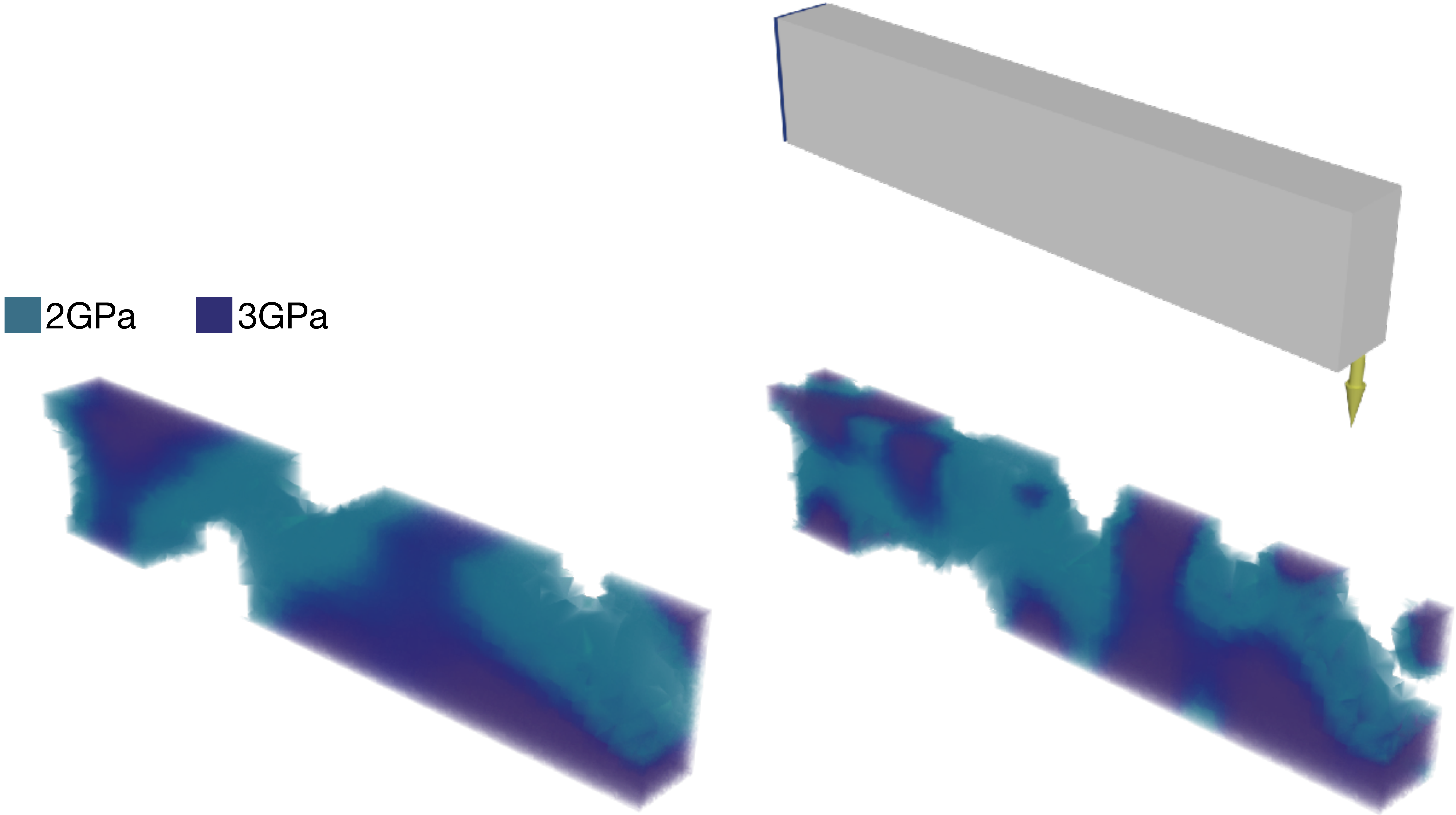
$$
\begin{aligned}
\min_{\boldsymbol{w}} \quad & \boldsymbol{u}^T \boldsymbol{K}(\boldsymbol{w})\boldsymbol{u} \\
\text{s.t.} \quad & m(\boldsymbol{w})/m_0 \leq m_{frac} \\
& \boldsymbol{K}(\boldsymbol{w})\boldsymbol{u} = \boldsymbol{F}
\end{aligned}
$$

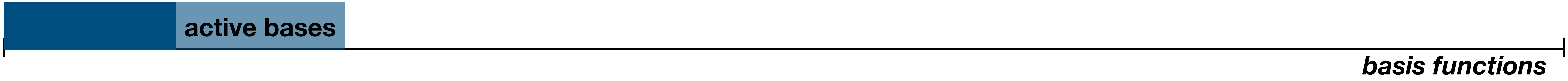


active bases

basis functions

Multi-material Topology Optimization Through Sliding Optimization Steps





active bases

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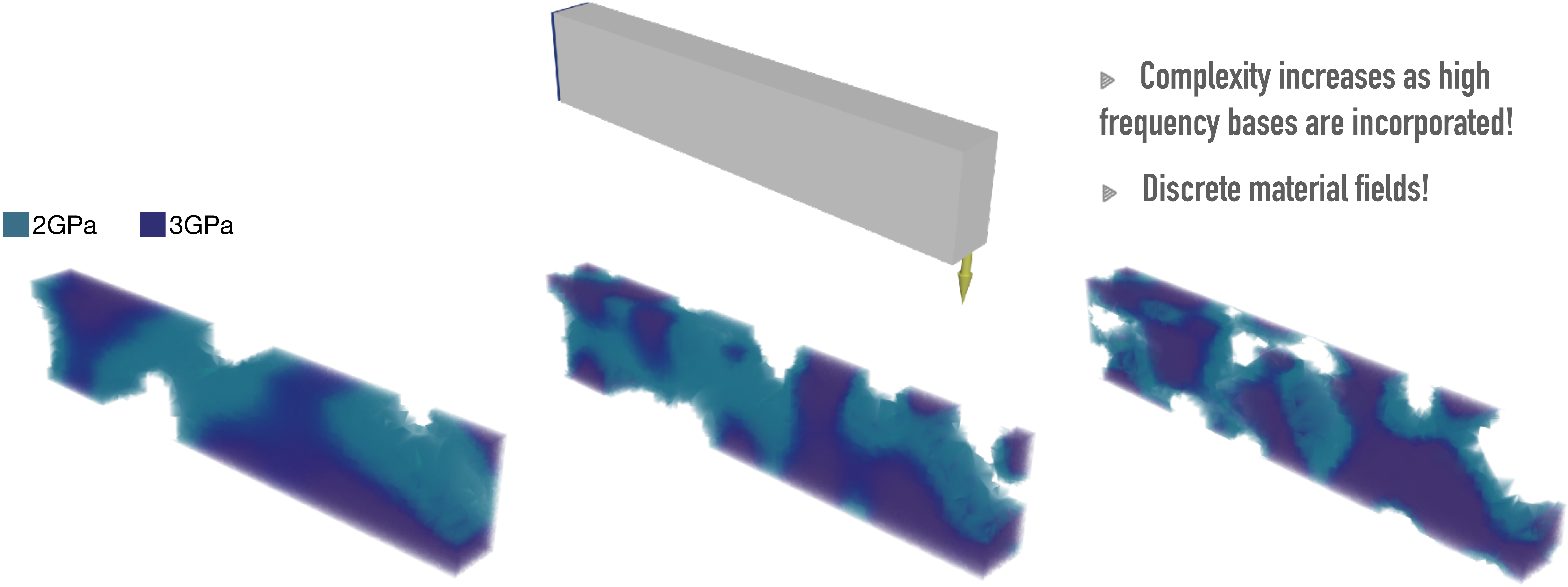
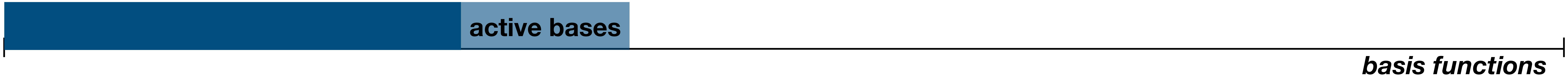
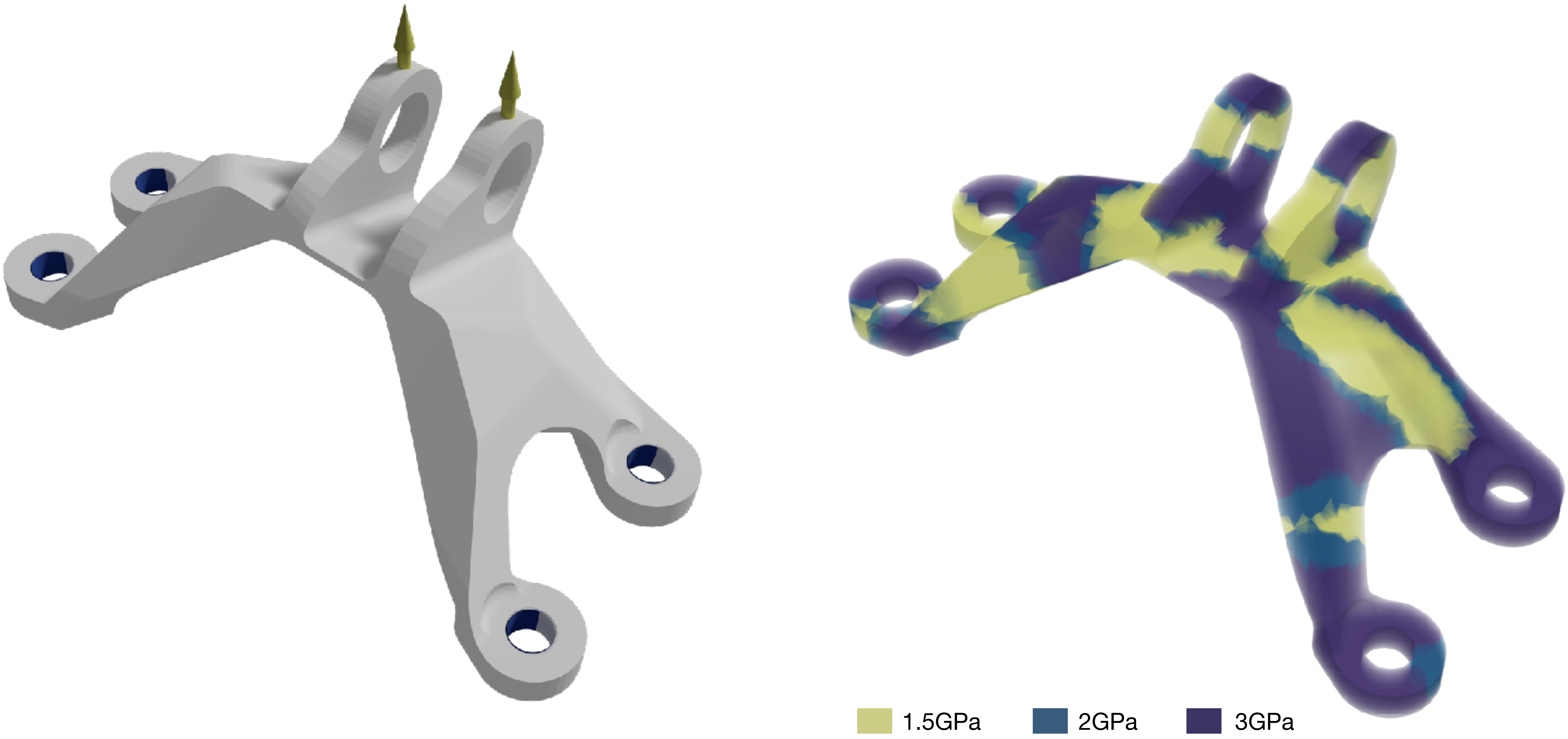
Multi-material Topology Optimization Through Sliding Optimization Steps



- Complexity increases as high frequency bases are incorporated!
- Discrete material fields!

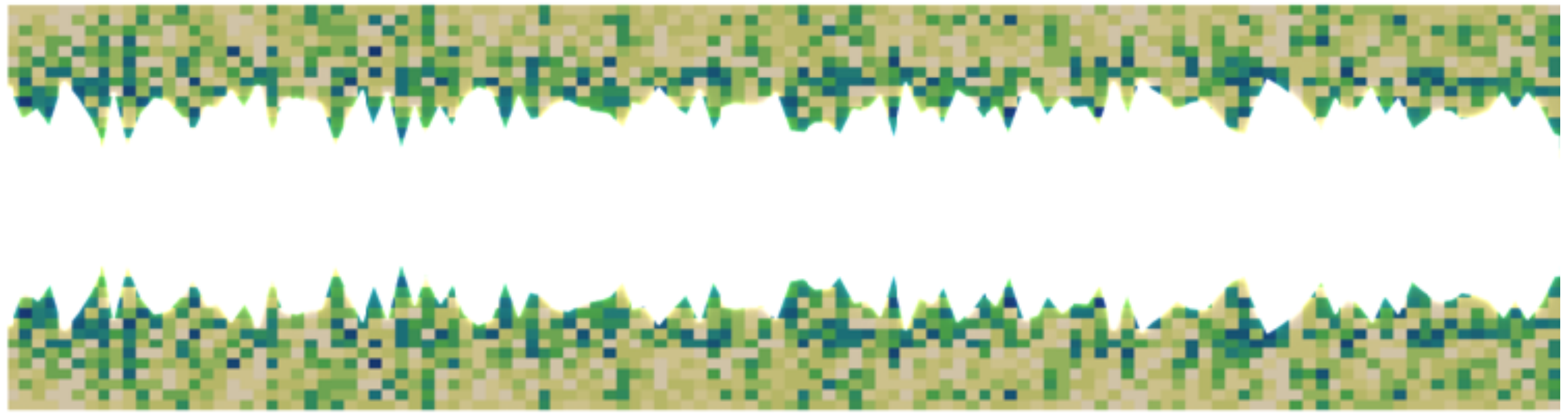


Material Distribution Optimization



Conventional VS Reduced Order

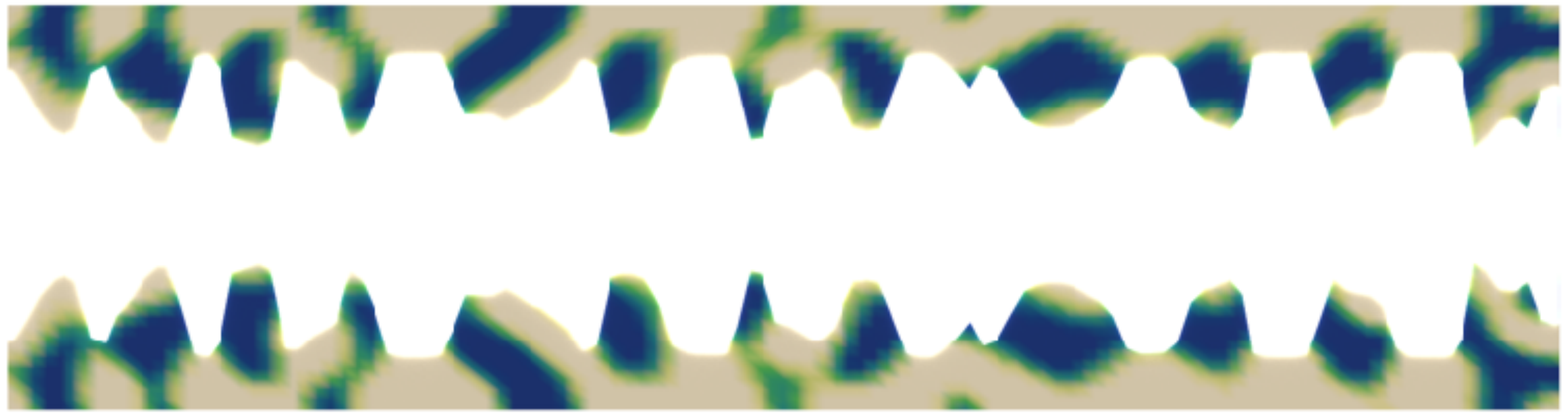
Optimize for each pixel



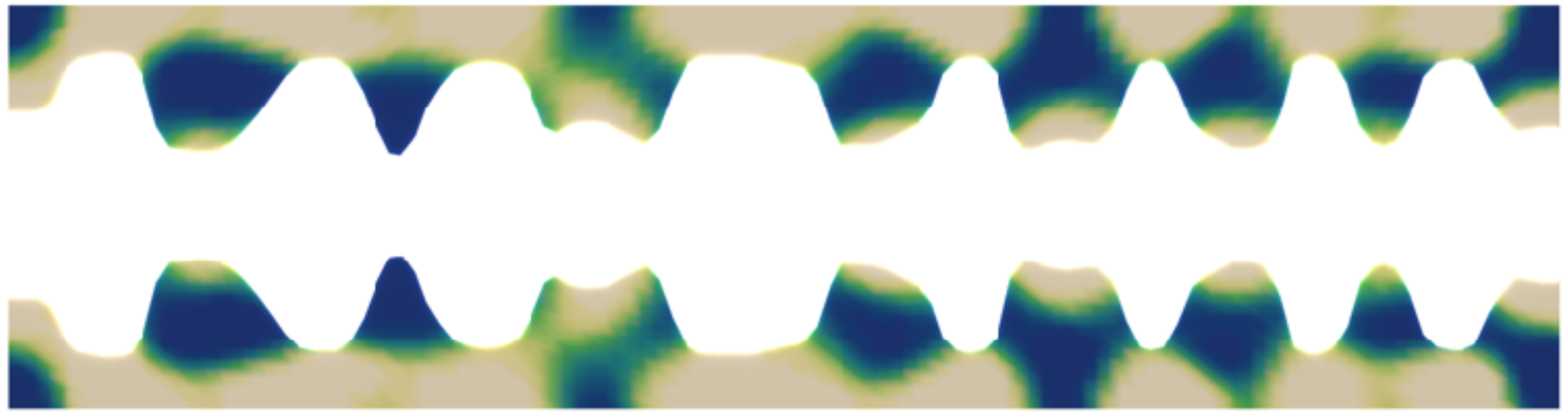
Objective: 252316 Time: 1945sec

Comparable Objective Values

Optimize for weights of the Laplacian basis (Ours)



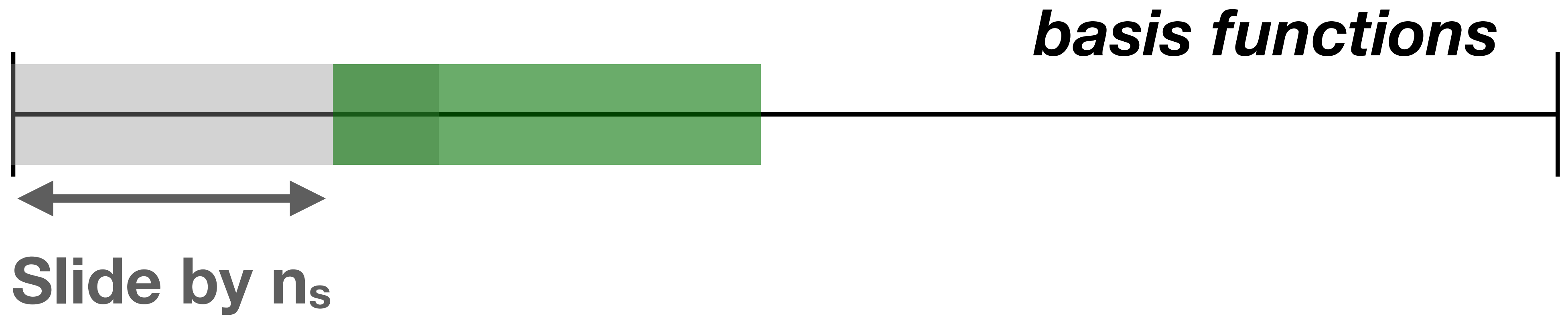
Sliding Basis — Objective: 253040 Time: 42sec

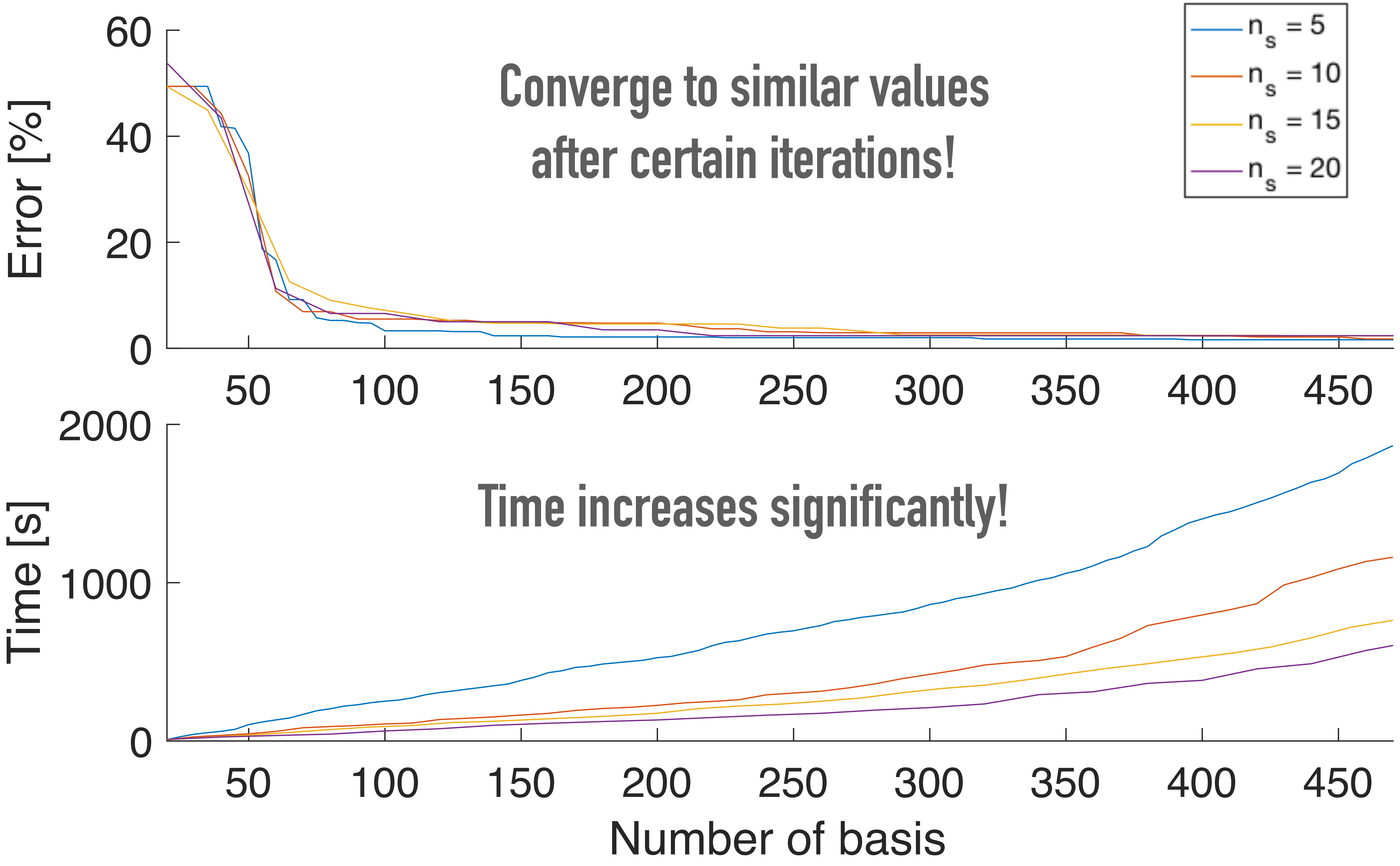


Fixed Basis — Objective: 287876 Time: 173sec

Reduced Order Faster

Effect of sliding amount, n_s





Converge to similar values after certain iterations!

Time increases significantly!

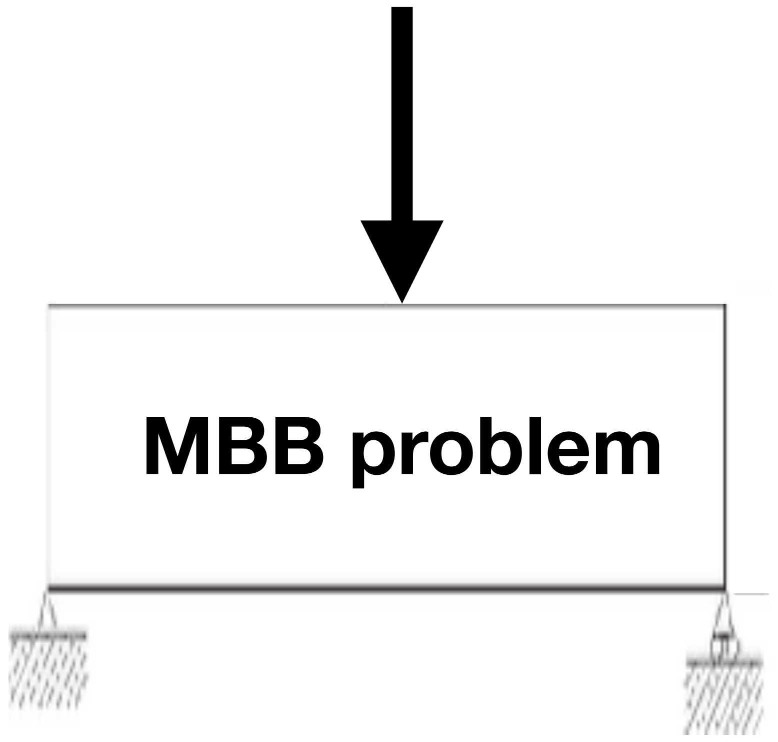
A Versatile Design Optimization Tool

The main **contributions** of the presented work:

- An optimization technique we call **sliding basis optimization** to **efficiently explore** parameterized design space
- **Practical material design method** with prescribed bounds using Laplacian basis
- **Enabling** optimization of material distributions for new applications coupled with **black-box analysis**

Recent Developments

Sliding basis topology optimization - a modular system



Sliding Basis Topology Optimization
Using **50 Bases**

Compliance objective: 302.2

Conventional Topology Optimization **(TOP88)**
Using **10k elements**

Compliance objective: 287.8

Goal is not to match the geometry but achieve comparable performance faster!!

Thank you!

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Supplementary

Acknowledgements

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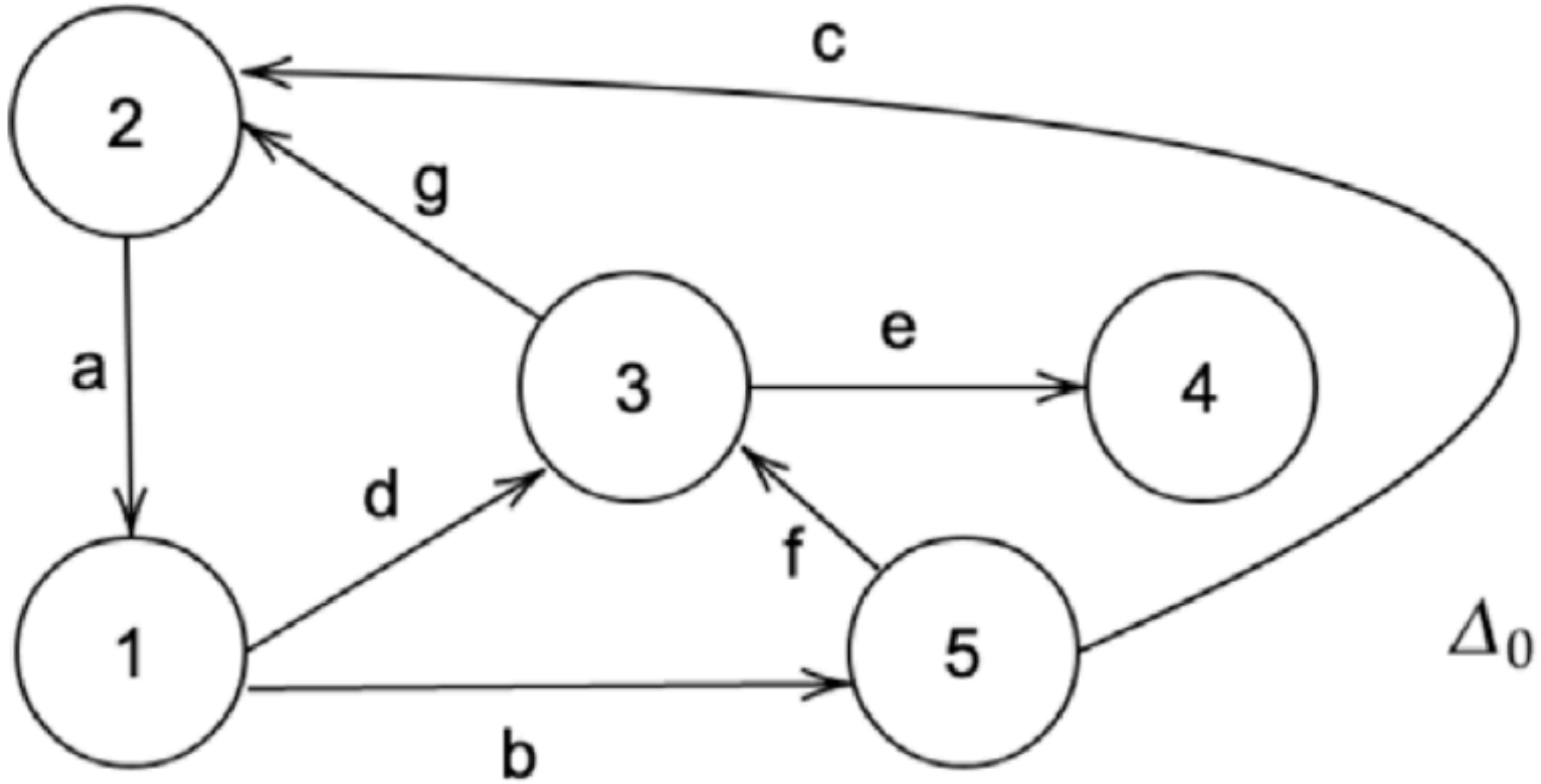
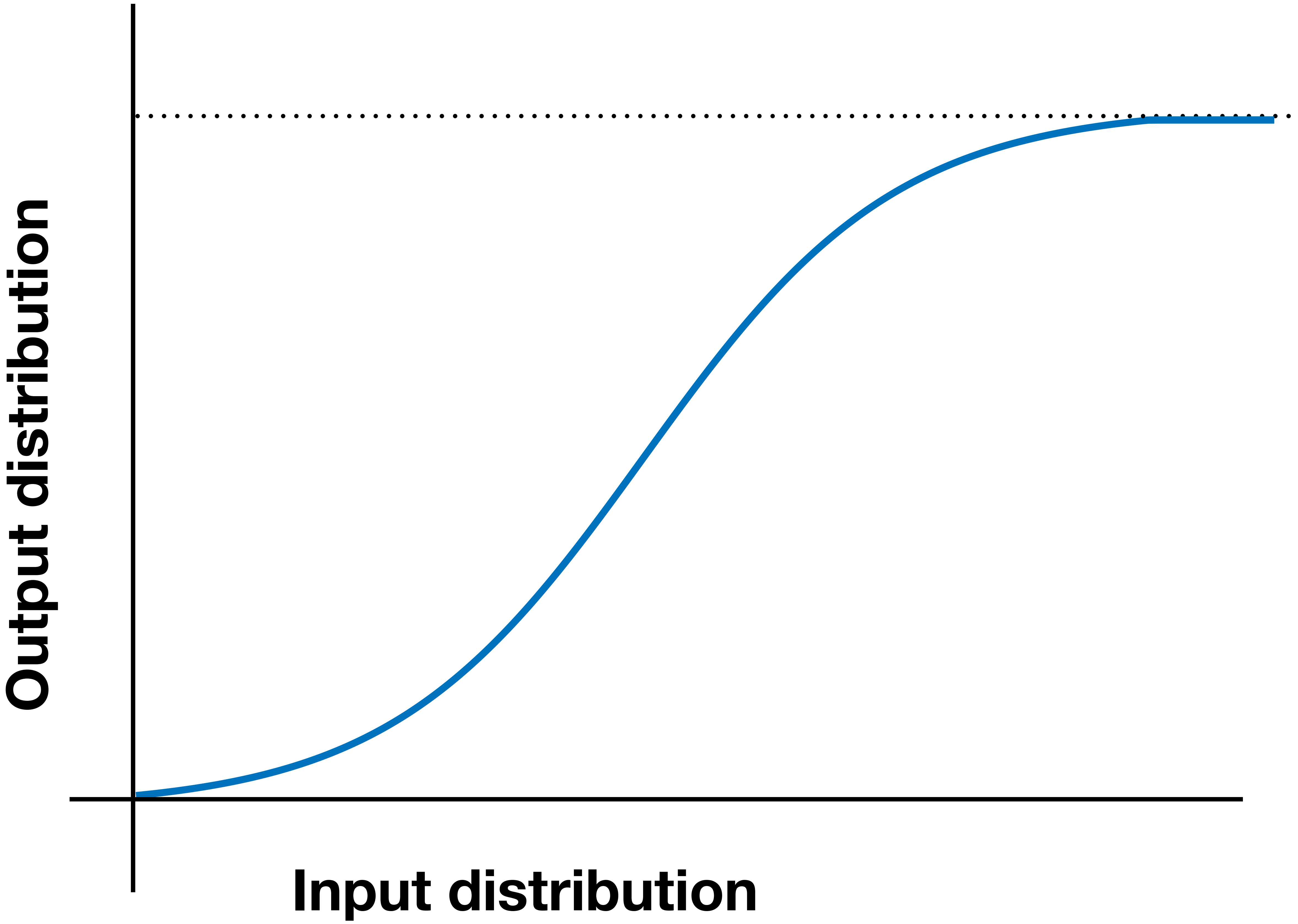
Graph Laplacian

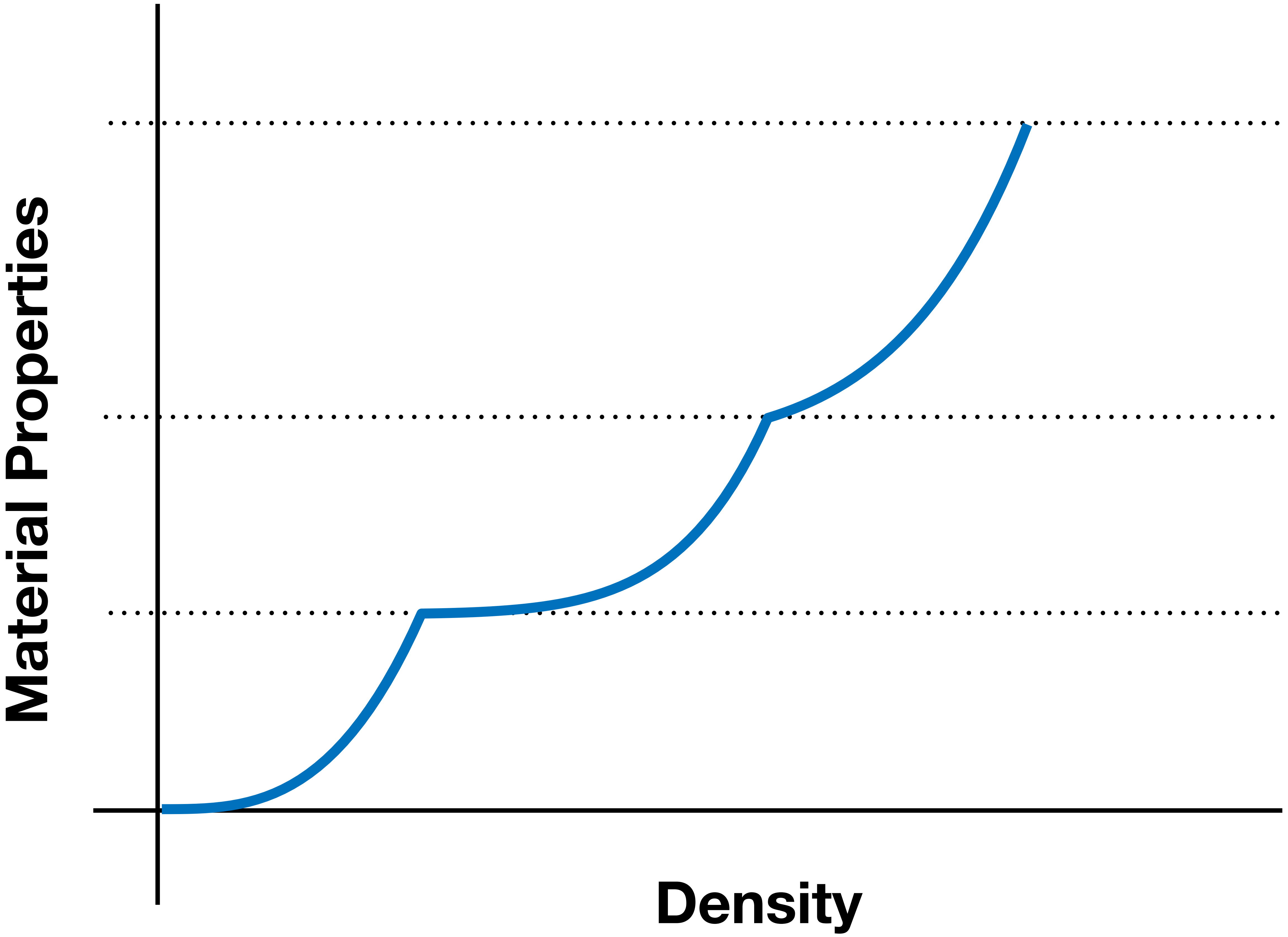


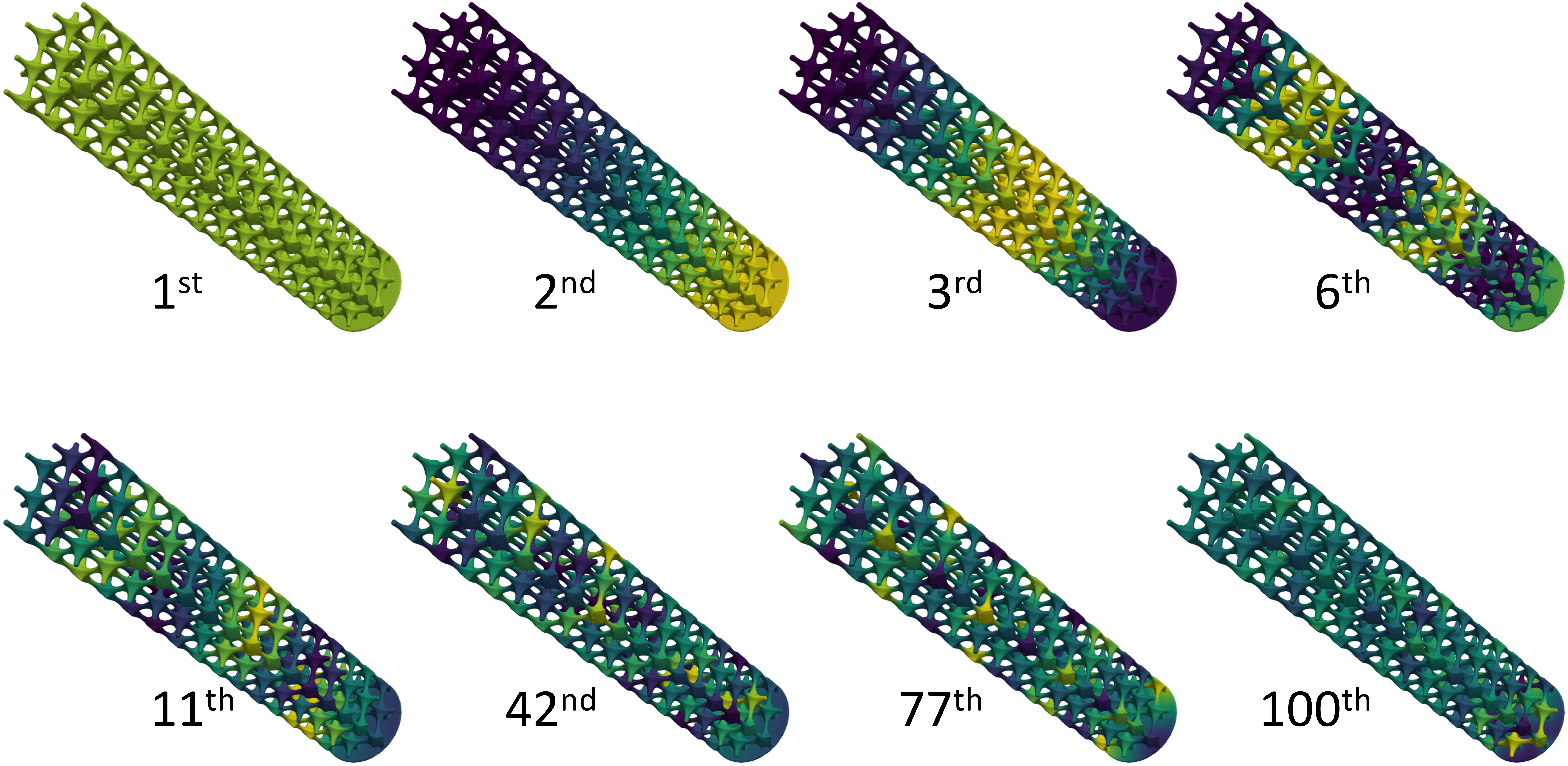
$$\partial_1 = \begin{pmatrix} -1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\Delta_0 = \partial_1 \, \partial_1^* = \begin{pmatrix} 3 & -1 & -1 & 0 & -1 \\ -1 & 3 & -1 & 0 & -1 \\ -1 & -1 & 4 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 \\ -1 & -1 & -1 & 0 & 3 \end{pmatrix}$$

Filters


Output distribution
Input distribution


Material Properties
Density


1st
2nd
3rd
6th
11th
42nd
77th
100th


0.254x10-2
burn rate [m/sec]
1.52x10-2

					Fixed Basis		Sliding Basis	
Thrust Profile	n_{opt}	n_s	n_{slides}	Total Basis	Time	Objective/Error	Time	Objective/Error
Constant Acceleration	20	15	14	230	1178s	349k/2.3%	288s	86k/1.1%
Constant Deceleration	50	40	7	320	4896s	867k/3.4%	621s	452k/2.7%
Two Step	20	15	7	125	191s	102k/1.1%	69s	217k/1.4%
Bucket	20	15	24	380	1006s	272k/1.8%	596s	272k/1.8%

Why not automatic differentiation?

- Numerical differentiation is already implemented and default option in many optimization software.
- But our approach can also work with automatic differentiation.
- One disadvantage of automatic differentiation is that it cannot be used with truly black-box components.